“A Study of The Problems of Heat Transfer in the flow of a non – Newtonian Second Order Fluid”

A

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Abstract

The study of non-Newtonian fluids (the fluids which do not obey the Newtonian law of viscosity) is of wide interest and significance as it deals with both the biological and non-biological fields. A non-Newtonian fluid is a fluid whose viscosity depends on the force applied (and sometimes time and temperature as well). Fluids like water and gasoline behave according to Newton's model, and are called Newtonian fluids but ketchup, blood, yogurt, gravy, pie fillings, mud and cornstarch paste don't follow the model. They are non-Newtonian fluids because doubling the speed that the layers slide past each other does not double the resisting force. It may less than double (like ketchup), or it may more than double (as in the case of quicksand and gravy). That's why stirring gravy thickens it, and why struggling in quicksand will make it even harder to escape.

Heat transfer is that science, which seeks to predict the energy transfer, which may take place between material bodies as a result of temperature difference. In the simplest of the terms, the discipline of heat transfer is concerned with only two things: temperature and flow of heat. Temperature represents the amount of thermal energy available, whereas heat flow represents the movement of thermal energy from one place to another place.

The theory of the Newtonian fluids gives a correction to the theory of perfect fluids, which is complete within terms of order one in c. If we neglect all the terms of order greater than two in c, then the simple fluid is called an incompressible second-order fluid.

In our present problem, we here study the flow pattern of an incompressible second-order fluid between two parallel infinite discs in the presence of transverse magnetic field when one is rotating (called rotor) and other is at rest (called stator). A uniform injection is applied to the stator forming the subject matter of the paper. The Rotor coincides with the plane $z = 0$ and the stator coincides with the plane $z = d$. Here the dimensionless parameters $\tau_1(\mu_2/pd^2)$, $\tau_2(\mu_2/pd^2)$ govern the effects of elastic-viscosity and cross–viscosity, while the effect of the injection are governed by a non- dimensional parameter k $(=w_0/2d\Omega)$ where w_0 is the uniform suction velocity (negative for injection).

This thesis has been divided into seven chapters.

1. The first chapter consists of the brief introduction of the topic along with the objectives, relevance, and role, importance of research work with literature review of

the study. This chapter also contain the MHD FLOW OF A SECOND-ORDER FLUID.

2. The second chapter highlights the Flow second-order fluid between two infinite discs in the presence of the presence of the magnetic field.
3. Third chapter discusses the Flow of a non-Newtonian second-order fluid over an enclosed torsinally Oscillating disc in the presence of the magnetic field.
4. The forth chapter discuss the Flow of a non-Newtonian second-order fluid between two enclosed torsionally oscillating discs in the presence of the magnetic field.
5. The fifth chapter contains the Heat transfer in the flow of a second-order fluid through a channel with porous wall under a transverse magnetic field .
6. The sixth chapter contains Heat transfer in the flow of a non-Newtonian second-order fluid over an enclosed torsion ally oscillating disc with uniform section and injection in the presence of the magnetic field.
7. The seventh chapter contains Heat transfer in the flow of a non-Newtonian second – order fluid second-order fluid between two enclosed torsion ally oscillating discs with uniform suction and injection in the presence of the magnetic field. And this chapter also discusses summary, conclusion including suggestions.

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CONTENTS

Chapter No. 1

Introduction

Introduction

The study of non-Newtonian fluids (the fluids which do not obey the Newtonian law of viscosity) is of wide interest and significance as it deals with both the biological and non-biological fields. A non-Newtonian fluid is a fluid whose viscosity depends on the force applied (and sometimes time and temperature as well). Fluids like water and gasoline behave according to Newton's model, and are called Newtonian fluids but ketchup, blood, yogurt, gravy, pie fillings, mud and cornstarch paste don't follow the model. They are non-Newtonian fluids because doubling the speed that the layers slide past each other does not double the resisting force. It may less than double (like ketchup), or it may more than double (as in the case of quicksand and gravy). That's why stirring gravy thickens it, and why struggling in quicksand will make it even harder to escape.

For some fluids (like mud or snow) we can push and get no flow at all until we push hard enough and the substance begins to flow like a normal liquid. This is what causes mudslides and avalanches.

Rheology is defined as the flow of fluids and deformation of solids under stress and strain. Rheometers are the instruments used to measure a material's rheological properties. Hook's low is probably the first recognizable law, which states that deformation, is proportional to the applied force. Newton considered the behaviour of an imaginary fluid when to fill all space, in which resistance to motion was proportional to what has variously been named rate of strain, rate of deformation, velocity strain or flow tensor d_{ij} and is known as Newton-Cauchy-Poisson law. Accordingly,

$$\tau_{ij} = p\delta_{ij} + 2\mu d_{ij} + \lambda d_m{}^m \delta_{ij}$$

where

$$d_{ij} = (u_{i,j} + u_{i,j})/2,$$

p is the pressure, μ and $= -2\mu/3)$ being material constants, also termed as coefficients of viscosity and δ_{ij} is kronecker's delta tensor. The fluids satistfying the relation (1.1), are called Newtonian fluids e.g. honey, glycerin and certain thick oils. For incompressible fluids the relation (1.1) becomes

$$\tau_{ij} = -p\delta_{ij} + 2\mu d_{ij}.$$

Although certain phenomena like skin-friction, form drag, separation, secondary flows etc., are successfully explained by this classical theory, but it has proved inadequate to explain the rheological properties of certain materials like paints, slurries, ceramics, melts poly-iso-butylene solution in the mineral oils or in tetralin, poly-methylmethacrylate solutions in the dimethyl-pthalate, rubber-toluene solutions etc. certain phenomena like Anmolous viscosity* the Weissenberg effect**, Merrington effect** and spinnability effect**** observed in these fluids could not be explain by the solutions of Navier-Stokes equations and therefore a basic search into the foundations of fluid dynamics had to be undertaken.

1.1 MAGNETOHYDRODYNAMICS:-

MHD is the study of the motion of the electrically conducting fluids in the presence of electric and magnetic fields. When a conducting fluid is under the influence of the electro-magnetic field, it behaves differently than without electromagnetic field. This is mainly because of Lorentz force, which is a cross product of electric field and magnetic field (Sir Flemming's right hand law). Even without the external electric field, flow pattern is altered due to the presence of strong magnetic field. Magnetic field and the motion of the conducting fluid particles generate electric current. This current and magnetic field interact with each other, and change the flow motion, with a chain reaction, all three fields (velocity, magnetic, electric) are interconnected and reveal very unique features.

One of the most popular applications of MHD is electricity generator. Since there is no mechanical friction whatsoever, this MHD generator has very high efficiency. However, the conducting fluid itself is usually heavy and harmful (for example, mercury), so its practical usage is limited. Nevertheless, as soon as 'super conducting' material is available, there is a hope that we can use sea-water as working fluid.

The other usage of MHD is turbulence control using electro-magnetic field. Since generally Lorentz force tends to suppress the fluid motion' we may be able to get some insights to utilize the Lorenz force in the turbulent drag reduction control.

*Anamolous viscosity[2)] or structural viscosity is the viscosity, the value of which changes with the driving pressure p in one apparatus or the rotational velocity Ω in the order.

**Weissenberg effect[3)] is the property of rising up of the fluid along the inner cylinder of the viscometer in steady shear or along a stirring rod.

***Merrington effect[4)] is the property of swelling of the liquid jet as it leaves the tube.

****Spinnability effect is the phenomena of spontaneous thread formation assessed by placing the tip of a rod in contact with the liquid surface and then drawing it away.

Dynamo action, the spontaneous generation of a magnetic field in a flow of conducting fluid, is supposed to be at the origin of the planets and stars magnetic fields. A lot of theoretical and numerical work has been devoted to this problem and it has been demonstrated Riga (Ponomarenko flow) and Karisruhe (Roberts flow).

This is mainly due to the fact that to build dynamo, one must use the liquid sodium, with all its incumbent dangers. Moreover, the dynamo will work only if the advection of the magnetic field and generation of the current are fast compared to dissipation- namely if the magnetic Reynolds number exceeds a critical value, which strongly depends on the geometry of the flow.

Application of MHD to natural events received a belated stimulus when astrophysicists came to realize how prevalent throughout the universe are conducting, ionized gases (plasmas) and significantly strong magnetic fields. The final implication was that MHD processes must dominate most areas of astrophysics. Larmor made the attractive suggestion that the magnetic fields of the sun and the other heavenly bodies might to be due to dynamo action, whereby the conducting material of the star acted as the armature and stator of a self-exciting dynamo. MHD is important in astrophysics because the enormous scale of events makes up for the smallness of the conductivities and magnetic fields.

The booms in post-war applied science soon affected MHD. Electromagnetic pumping of liquid metal coolants in nuclear reactors became standard practice and electromagnetic pumping, string and levitation (to avoid contamination) were exploited in the metallurgical industries.

It seems that the first attempt to study the problem of magnetohydrodynamics was made by Faraday[5)] (1832). Thereafter Williams[6)], Stewartson[7)], Bhatnagar[8)] And Andersson[9),] Charyulu[10)], Helmy[11)], Singh and Thakur[12)] developed the field successfully.

1.2 HEAT TRANSFER

Heat transfer is that science, which seeks to predict the energy transfer, which may take place between material bodies as a result of temperature difference. In the simplest of the terms, the discipline of heat transfer is concerned with only two things: temperature and flow of heat. Temperature represents the amount of thermal energy available, whereas heat flow represents the movement of thermal energy from one place to another place.

On a microscopic scale, thermal energy is related to the kinetic energy of the molecules. The greater a material's temperature, The greater the thermal agitation of its constituent molecules (manifested both in linear motion and vibrational modes). It is natural for regions containing greater molecular kinetic energy to pass this energy to regions with less kinetic energy.

Several material properties serve to modulate the heat transferred between two regions at differing temperatures. Examples include thermal conductivities, specific heats, material densities, fluid velocities, fluid viscosities, surface emissivities, and more. Taken together, these properties serve to make the solution of many heat transfer problems an involved process.

Heat transfer mechanisms can be grouped into three bond categories:

(i) CONDUCTION:

Regions with greater molecular kinetic energy will pass their thermal energy to regions with less molecular energy through direct molecular collisions, a process known as conduction. In metals, conduction-band electrons also carry a significant portion of the transported thermal energy.

(ii) CONVECTION:

When heat conducts into a static fluid it leads to a local volumetric expansion. As a result of gravity-induced pressure gradients, the expanded fluid parcel becomes buoyant and displaces, thereby transporting heat by fluid motion (i.e. convection) in addition to conduction. Such heat-induced fluid motion in initially static fluids is known as free convection.

For cases where the fluid is already in motion, heat conducted into the fluid will be transported away chiefly by fluid convection. These cases, known as forced convection, require a pressure gradient to drive the fluid motion, as opposed to a gravity gradient to induce motion through buoyancy.

(iii) RADIATION:

All materials radiate thermal energy in amounts determined by their temperature, where the energy is carried by photons of light in the infrared and visible portions of the electromagnetic spectrum. When temperatures are uniform, the radiative flux between objects is in equilibrium and no net thermal energy is exchanged. The balance is upset when temperature is not uniform, and thermal energy is transported from surfaces of higher to surfaces of lower temperature.

Some of the major contributions in the field or heat-transfer are due to Tao[13)], Cess [14)], Riley [15)], Sastry[16)], Bestman[17)], Kasture[18)], Sharma, Sunil Pal[19)].

1.3 RHEOLOGICAL EQUATION OF STATE OR CONSTITUTIVE EQUATION

It expresses the relationship between stress and rate of strain.

On the basis of large amount data, which could not be explained on the basis of linear assumption, it is felt that linearity is too drastic an assumption and one should explore mathematically the consequences of more general functional relationship than that for a Newtonian fluid. The availability of high speed computers have rendered possible the solution of non-linear equations involved. This development has lead to the growth of the subject matter of non-linear mechanics.

Non-linearity in the equation (1.1) has been attempted in a number of ways. The first generalization consisted in taking λ and μ occurring in this equation as functions of three invariant of strain-rate tensor d_{ij}. Non-linearity in stress-strain-rate law is thus introduced through the material constants. Other generalizations were obtained by including terms corresponding to elastic and plastic properties of the material and micro-rotational inertia and micro-rotational effects. These different generalizations gave rise to the study of the fluids called plastics, pseudo-plastics, Bingham palastic, dilatants materials, non-Newtonian fluids (including the rheopectic and thixotropic fluids). All these fluids along with their constitutive equations are treated in details in references[20-30].

1.4 Rivlin-Ericksen Fluids:-

Starting with different premises, Rivlin and Ericksen[31] enunciated a theory for elastic-viscous fluids, known as Rivlin-Ericksen theory. The constitutive equation in this case is[32]

$$\tau = -pI + \phi_1 A + \phi_2 B + \phi_3 A^2 + \phi_4 B^2 + \phi_5 (AB+BA) + \phi_6(A^2B+BA^2) + \phi_7(AB^2 + B^2A) + (A^2B^2 + B^2A^2), \qquad (1.2)$$

Where I is a unit matrix and ϕ's are polynomial functions in the various invariants trA, trA^2, trA^3, trB, trB^2 , trAB, $trAB^2$, trA^2B and trA^2B^2.

$$A= 2[d_{ij}] = [u_{i,j} + u_{i,j}],$$

$$B= 2[e_{ij}] = [a_{i,j} + a_{i,j}], 2u^{m}{}_{,i}u_{m,j},$$

u_{ij} being velocity vector.

From expression (1.2), it is evident that if we put ϕ_2, ϕ_4, ϕ_5, _ _ _ _ _ ϕ_8 as zero; we obtain equation for Reiner-Rivlin fluids. Thus Reiner-Rivilin fluids are particular cases of Rivlin-Ericksen fluids. Some of the work in this fields is reviews in refrences[33-35)]

As far as we know, this theory is adequate, to describe the steady viscometric flows and some of the time dependent behaviour. However, it does to explain the phenomena of gradual stress relaxation after the ceasation of the motion, which is a commonly observed elastic-viscous effect.

1.5 SECOND-ORDER FLUIDS:-

A Theory of more general type of incompressible fluid was put forward by Green et. AI.[36)], Coleman and Noll[37)]. The Theory is based on the hypothesis that the stress is a function of the deformation gradient, that is the stress at the material point depends only on the previous history of the deformation gradient. The materials obeying this theory are termed as simple materials by Noll[37)].

An incompressible simple fluid is an incompressible simple material if it possesses the property that all local states with the same mass density are intrinsically equivalent in response. For a given history g(s) a retarded history $g_c(s)$ can be defined as:

$$g_c(s) = g(s),\ 0 \le s \le \infty,$$

(1.3)

where c is the retardation factor $0 \le c < 1$. taking into consideration, this definition of retarded history and assuming that the stress is more sensitive to recent deformation than to deformations which occurred in the distant past, Coleman and Noll[37)] proved that the theory of the simple fluids yields the theory of perfect fluids (in which deviatoric stress is independent of strain-rate) for c proved that the theory of the simple fluids yields the theory of perfect fluids (in which deviatoric stress is independent of strain-rate) for $c \to 0$ and yields the theory of the Newtonian fluids (in which deviatoric stress is linearly proportional to deviatoric strain-rate) as the next approximation.

The theory of the Newtonian fluids gives a correction to the theory of perfect fluids, which is complete within terms of order one in c. If we neglect all the terms of order greater than two in c, then the simple fluid is called an incompressible second-order fluid. The constitutive equation of non-Newtonian second-order fluid is

$$\tau_{ij} = -p\delta_{ij} + 2\mu_1 d_{ij} + 2\mu_1 e_{ij} + 2\mu_3 c_{ij}$$

(1.4)

On taking $\mu_2 = 0$, we get the constitutive equation for Reiner-Rivlin visco-inelastic fluid as

$$\tau_{ij} = -p\delta_{ij} + 2\mu d_{ij} + 4\mu_3 c_{ij} \quad (1.5)$$

where

$$d_{ij} = {}^1/_2 [u_{i,j} + u_{i,j}],$$

$$e_{ij} = {}^1/_2\ [a_{i,j} + a_{i,j}],\ u^m{}_{,i} u_{m,j},$$

$c_{ij} = d_{im} d^{m}_{j}$.

p is the indeterminate hydrostatic pressure; τ_{ij} is the stress-tensor; and a_i are the velocity and acceleration vector and μ_1, μ_2, μ_3 are called the coefficient of Newtonian-viscosity, the coefficient of elastic-viscosity and the coefficient of cross-viscosity respectively.

Rivlin, Noll, Coleman, Markowitz and others[38-41)] have solved elementary flow problems (steady as well as unsteady in nature) for these fluids. Some evidence favoring the Weissenberg effect etc. were given by Roberts and others[42,43)], Coleman, Noll, Ericksen and Markowitz contended that the most general type of fluid is characterized by three functions of the rate of shear[38,41,44)].

Ting[45)] has taken positive values of the elastic-viscosity but later it was confirmed that it should be taken as negative[41,46)]. The problems concerning the behaviour of the second-order fluids have also been discussed by Langlois[47)], Srivastava[48,48A)], Sharma[49)], Gupta[50)], Sharma[51)], Bhatia[52)] , Sharma[53)] , Prakash[54)] , Gupta[55)] , Singh[55A)] , Smit[56)] , Rita Chaudhary and Alok Das[57)].

1.6 Basic Equation

The governing equations, which will be used in the problems, are as follows:

1. **Equation of Continuity:**

The law of conservation of mass states that fluid mass can be neither created nor destroyed. The equation of continuity aims at expressing the law of conservation of mass in a mathematical form.

Thus in continuous motion, the equation of continuity expresses the fact, the increase in the mass of fluid within any closed surface drawn in the fluid in any time must be equal to the excess of the mass that flows in over the mass that flows out.

$$\partial\rho/\partial t + (\rho u)_{,i} = 0$$

Where u^i and ρ are respectively the velocity vector and density of the fluid. For incompressible fluids this equation reduce to

$$U^i_{,i} = 0 \qquad (1.7)$$

2. Momentum Equation:

These equations are based on the Newton's law of motion, which continues to be the basis of all continuum mechanics except relativistic mechanics.

$$\rho(\partial u_i/\partial t + u_m u_{i,m}) = Pf_i + \tau^m{}_{i,m} \qquad (1.8)$$

Where F is the impressed force per unit mass of fluid and $\tau^m{}_i$ the stress tensor. The Momentum equation for no extraneous force is simply

$$\rho(\partial u_i/\partial t + u_m u_{i,m}) = \tau^m{}_{i,m}$$

(1.9)

3. Equation of Energy:

This equation is based on the first law of Thermodynamics. For incompressible fluid the energy balance is determined by the internal energy, the conduction of the heat, the convection of the heat with the stream and the generation of the heat through friction. In a compressible fluid there is an additional term due to the work of expansion (or compression) when the volume is changed. In all cases radiation may also be present, but its contribution is small at moderate temperatures, and we shall neglect it completely.

$\rho c_v(\partial T/\partial t + u^m T,_m) = kg^{ig}T_{,IJ} + \phi,$

(1.10)

Where T is the temperature, c_v the specific heat at constant volume, k the thermal conductivity, g^{ij} the associate of metric tensor g_{ij} and ϕ, the dissipation function is given by

$\phi = \tilde{\tau}^i_j d^j_{i,}$

$\tilde{\tau}^i_j$ is the mixed deviatoric stress tensor.

4. The equations of electromagnetic field:

Maxwell's equations:

div **B** = 0,

(1.11)

div **D** = $\rho_{e,}$

(1.12)

Curl **E** = -∂**B**/∂t,

(1.13)

Curl **H** =**J**+$\partial \boldsymbol{D}/\partial t$.

(1.14)

Ohm's law:

J = $\boldsymbol{\sigma}$ **(E+VxB)**+ ρ_e **V,**

(1.15)

Where

B = $\boldsymbol{\mu_e}$ **H,**

$\mathbf{D} = \epsilon_e \mathbf{E}$,

Also the Lorenz force is given by

$$\rho\mathbf{F} = \mathbf{J} \times \mathbf{B} + \rho_e\mathbf{E}$$

(1.16)

Where **B** is the electromagnetic induction, **E**is the electric field, **H** is the magnetic field, **D**the density of the electric displacement, **J**is the electric current density, ρ_e the electric charge density, ϵ_e the di-electirc constant, μ_e the magnetic permeability and σ the electric conductivity.

Thus the equation of energy for incompressible MHD fluid is

$$\rho c_v(\partial T/\partial t + u^m T,_m) = \mathbf{J}^2/\sigma + kg^{ig}T_{,IJ} + \phi$$

(1.17)

And the equation of motion will become

$$\rho(\partial u_i/\partial t + u^m u_{i,m}) = \mathbf{J} \times \mathbf{B} + \tau^m{}_{i,m}$$

(1.18)

APPENDIX

The transformation of the basic equations, in terms of the Cartesian and cylindrical polar co-ordinates in physical component's form is as follows:

(A)CARTESIAN SYSTEM (x,y,z)

Equation of continuity (1.7) transforms to

$$(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z) = 0$$

(A.1)

and momentum equations take the form

$$\rho\ Du/Dt = (\partial \tau_{xx}/\partial x) + (\partial \tau_{yx}/\partial y) + (\partial \tau_{zx}/\partial z) + \rho F_x$$ (A.2)

$$\rho\ Dv/Dt = (\partial \tau_{xy}/\partial x) + (\partial \tau_{yy}/\partial y) + (\partial \tau_{zz}/\partial z) + \rho F_y \qquad (A.3)$$

$$\rho\ Dw/Dt = (\partial \tau_{xz}/\partial x) + (\partial \tau_{yz}/\partial y) + (\partial \tau_{zz}/\partial z) + \rho F_z \qquad (A.4)$$

where u, v, w; $\tau_{xx}, \tau_{yy}, \tau_{zz}, \tau_{xy}, \tau_{xz}, \tau_{yz}, \rho F_x, \rho F_y, \rho F_z$ are the physical components of the velocity vector, the stress tensor and the Lorenz force vector respectively while D/Dt represents the material derivative given by

$$D/Dt = (\partial \tau/\partial t) + (u\partial/\partial x) + (v\partial/\partial y) + (w\partial/\partial z)$$

$$= (u\partial/\partial x) + (v\partial/\partial y) + (w\partial/\partial z) \text{ for steady flow.}$$

The constitutive equations (1.5) and (1.4) in terms of the physical components take the following form:

$$\tau_{xx} = -p + 2\mu_1 d_{xx} + 4\mu_3 c_{xx},$$

$$\tau_{yy,} = -p + 2\mu_1 d_{yy} + 4\mu_3 c_{yy},$$

$$\tau_{zz,} = -p + 2\mu_1 d_{zz} + 4\mu_3 c_{zz},$$

$$\tau_{xy} = 2\mu_1 d_{xy} + 4\mu_3 c_{xy},$$

$$\tau_{xy} = 2\mu_1 d_{yz} + 4\mu_3 c_{yz},$$

$$\tau_{xz} = 2\mu_1 d_{zx} + 4\mu_3 c_{zx}; \qquad (A.5)$$

$$\tau_{xx} = -p + 2\mu_1 d_{xx} + 2\mu_2 e_{xx} + 4\mu_3 c_{xx},$$

$$\tau_{yy,} = -p + 2\mu_1 d_{yy} + 2\mu_2 e_{yy} + 4\mu_3 c_{yy},$$

$$\tau_{zz,} = -p + 2\mu_1 d_{zz} + 2\mu_2 e_{zz} + 4\mu_3 c_{zz},$$

$$\tau_{xy} = 2\mu_1 d_{xy} + 2\mu_2 e_{xy} + 4\mu_3 c_{xy},$$

$\tau_{xy} = 2\mu_1 d_{yz} + 2\mu_2 e_{yz} + 4\mu_3 c_{yz}$,

$\tau_{xz} = 2\mu_1 d_{zx} + 2\mu_2 e_{zx} + 4\mu_3 c_{zx}$; (A.6)

where d_{xx} etc. e_{xx} etc. c_{xx} etc. and p are physical components of the symmetric tensors $d_{ij,}$ e_{ij}, c_{ij} and pressure in the directions indicates by second suffix. For steady flow – etc. are given by:

$d_{xx} = (\partial u/\partial x)$, $d_{yy} = (\partial v/\partial y)$ $d_{zz} = (\partial w/\partial z)$

$d_{xy} = (1/2)\, d(\partial u/\partial y + \partial v/\partial x)$, $d_{yz} = (1/2)(\partial v/\partial z + \partial w/\partial y)$,

$d_{zx} = (1/2)\, d(\partial w/\partial x + \partial u/\partial z)$.

(A.7)

Now

$e_{ij} = {}^1/_2\,[a_{i,j} + a_{i,j}] + u^m{}_{,i} u_{m,j}$,

where acceleration $a_i = Du_i/Dt = \partial u_i/\partial t + u^m{}_{,i} u_m$ and for steady flow

$a_i = u\partial u/\partial x + v\partial u/\partial y + w\partial u/\partial u + u^m{}_{,i} u_m$ Therefore

$e_{ij} = D(d_{ij})/Dt + (d^m{}_{,i} u_{m,i} + d^m{}_{,i} u_{m,i})$.

Thus

$e_{xx} = \{D(d_{xx})/Dt\}\; 2\{(\partial u/\partial x))d_{xx} + (\partial v/\partial x)d_{xx} + (\partial v/\partial x)d_{yx} + (\partial w/\partial x)d_{zx}\}$

for steady flow

$e_{xx} = (u\partial/\partial x + v\partial/\partial y + w\partial/\partial z)d_{xx} + 2\{(\partial u/\partial x)d_{xx} + (\partial v/\partial x)d_{yx} + (\partial w/\partial x)d_{zx}\}$,

$e_{yy} = (u\partial/\partial x + v\partial/\partial y + w\partial/\partial z)d_{yy} + 2\{(\partial u/\partial y)d_{xy} + (\partial v/\partial y)d_{yy} + (\partial w/\partial y)d_{zy}\}$,

$e_{zz} = (u\partial/\partial x + v\partial/\partial y + w\partial/\partial z)d_{zz} + 2\{(\partial u/\partial z)d_{xz} + (\partial v/\partial x)d_{yz} + (\partial w/\partial z)d_{zz}\}$,

$e_{xy} = (u\partial/\partial x + v\partial/\partial y + w\partial/\partial z)d_{xy} + (\partial u/\partial x + \partial u/\partial y)d_{xy} + (\partial v/\partial x)d_{yy} + (\partial v/\partial x)d_{yy} + (\partial w/\partial x)d_{yz} + (\partial w/\partial y)d_{zx} + (\partial u/\partial y)d_{xx}$,

$e_{yz} = (u\partial/\partial x + v\partial/\partial y + w\partial/\partial z)d_{yz} + (\partial u/\partial y)d_{yz} + (\partial u/\partial y)d_{xz} + (\partial v/\partial y)d_{yz} + (\partial w/\partial y)d_{zz} + (\partial u/\partial z)d_{xy} + (\partial v/\partial z)d_{yy} + (\partial w/\partial z)d_{zy}$,

$e_{zx} = (u\partial/\partial x + v\partial/\partial y + w\partial/\partial z)d_{zx} + (\partial u/\partial z)d_{xx} + (\partial v/\partial z)d_{yx} + (\partial w/\partial z)d_{zx} + (\partial u/\partial x)d_{zx} + (\partial v/\partial x)d_{yz} + (\partial v/\partial z)d_{yy} + (\partial w/\partial x)d_{zz}$,

(A.8)

$c_{xx} = d_{xx}^2 + d_{xy}^2 + d_{xz}^2$,

$c_{yy} = d_{xy}^2 + d_{yy}^2 + d_{zy}^2$,

$c_{zz} = d_{xz}^2 + d_{yz}^2 + d_{zz}^2$,

$c_{xy} = d_{xx}\, d_{xy} + d_{xy}\, d_{yy} + d_{xz}\, d_{zy}$,

$c_{xy} = d_{xx}\, d_{xy} + d_{xy}\, d_{yy} + d_{xz}\, d_{zy}$,

$c_{yz} = d_{xz}\, d_{xy} + d_{yz}\, (d_{yy} + d_{zz})$,

$c_{zx} = d_{xy}\, d_{yz} + d_{xz}\, (d_{xx} + d_{zz})$,

(A.9)

The energy equation (1.10) transforms to --

$$\rho c_v\,(DT/Dt) = k\,\nabla^2 T + \phi,$$

(A.10)

Where

$$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$$

And

$\phi = \tilde{\tau}_{xx}d_{xx}+\tilde{\tau}_{yy}d_{yy}+\tilde{\tau}_{zz}d_{zz}+2(\tilde{\tau}_{xy}d_{xy}+\tilde{\tau}_{xz}d_{xz}+\tilde{\tau}_{yz}d_{yz})$,

$\tilde{\tau}_{xx}$ etc. are the deviatoric stress tensors.

(B) <u>Cylendrical Polar System (r,θ,z)</u>

In this system equation of continuity (1.7) takes he form

$$\partial u/\partial r + u/r + (1/r)\, \partial v/\partial \theta + \partial w/\partial z = 0 \quad \text{(B.1)}$$

and the momentum equations (1.8) take the form

$$p(Du/Dt - v^2/r) = \partial \tau_{rr}/\partial r + (1/r)(\partial \tau_{r\theta}/\partial r\, \theta) + (1/r)(\tau_{rr}/\tau_{\theta\theta}) + \partial \tau_{rz}/\partial z \quad \text{(B.2)}$$

$$p(Dv/Dt + uv/r) = \partial \tau_{r\theta}/\partial r + (1/r)(\tau_{\theta\theta}/\partial \theta) + \partial \tau_{z\theta}/\partial z + 2\tau_{r\theta/}r, \quad \text{(B.3)}$$

$$p(Dw/Dt) = \partial \tau_{rz}/\partial r + (1/r)(\partial \tau_{\theta z}/\partial \theta) + \partial \tau_{zz}/\partial z + \tau_{rz}/r, \quad \text{(B.4)}$$

where

$D/Dt \equiv \partial/\partial t + u\partial/\partial r + (v/r)(\partial/\partial \theta) + e\partial/\partial z$,

u, v, w and $\tau_{rr,}$ $\tau_{\theta\theta}$, τ_{zz}, $\tau_{r\theta}$, $\tau_{\theta z}$, τ_{zr} are the physical components of the velocity vector and stress tensor respectively.

The constitutive equations (1.5) and (1.4) in terms of the physical components of the stress tensor are given by the following set of equation:

$$\tau_{rr} = -p + 2\mu_1 d_{rr} + 4\mu_3 c_{rr},$$

$$\tau_{yy,} = -p + 2\mu_1 d_{yy} + 4\mu_3 c_{\theta\theta},$$

$\tau_{zz,} = -p + 2\mu_1 d_{zz} + 4\mu_3 c_{zz}$,

$\tau_{r\theta} = 2\mu_1 d_{r\theta} + 4\mu_3 c_{zz}$,

$\tau_{\theta z} = 2\mu_1 d_{\theta z} + 4\mu_3 c_{\theta z}$,

$\tau_{\theta z} = 2\mu_1 d_{\theta z} + 4\mu_3 c_{\theta z}$,

$\tau_{zr} = 2\mu_1 d_{zr} + 4\mu_3 c_{zr}$;

(B.5)

$\tau_{rr} = -p + 2\mu_1 d_{rr} + 2\mu_2 e_{rr} + 4\mu_3 c_{rr,}$

$\tau_{\theta\theta,} = -p + 2\mu_1 d_{\theta\theta} + 2\mu_2 e_{\theta\theta} + 4\mu_3 c_{\theta\theta}$,

$\tau_{zz,} = -p + 2\mu_1 d_{zz} + 2\mu_2 e_{zz} + 4\mu_3 c_{zz}$,

$\tau_{r\theta} = 2\mu_1 d_{r\theta} + 2\mu_2 e_{r\theta} + 4\mu_3 c_{r\theta}$,

$\tau_{\theta z} = 2\mu_1 d_{\theta z} + 2\mu_2 e_{\theta z} + 4\mu_3 c_{\theta z}$,

$\tau_{zr} = 2\mu_1 d_{zr} + 2\mu_2 e_{zr} + 4\mu_3 c_{zr}$;

where physical components of tensors d_{ij}, e_{ij} and c_{ij} are

$d_{rr} = \partial u/\partial r$, $d_{\theta\theta} = (1/r)(u + \partial v/\partial\theta)$, $d_{rr} = \partial w/\partial z$

The energy equation (1.10) transforms to

where

$d_{r\theta} = (1/2)\{1/r)(\partial u/\partial\theta)/v/r + \partial v/\partial r,$

$d_{\theta z} = (1/2)\{\_\partial v/\partial z + (1/r)\,\partial w/\partial\theta)\}$,

$d_{zr} = (1/2)(\partial u/\partial z + \partial w/\partial r)$.

(B.7)

$d_{zr} = (1/2)(\partial u/\partial z + \partial w/\partial r)$.

(B.7)

$e_{rr} = \partial^2 u/\partial r\partial t + 2(\partial u/\partial r)^2 + (\partial v/\partial r)^2 + (\partial w/\partial r)^2 + u\partial^2 u/\partial r + (v/r)(\partial^2 u/\partial r\partial\theta) - (v/r^2)(\partial u/\partial\theta) + (1/r)(\partial u/\partial\theta)(\partial v/\partial r) + (\partial w/\partial r)(\partial u/\partial z) + w\partial^2 u/\partial r\partial z - (2v/r)(\partial v/\partial r) + v^2/r^2$,

$e_{\theta\theta} = (\partial/\partial t)\{(1/r)(\partial v/\partial\theta) + u/r\} + (1/r^2)(\partial u/\partial\theta)^2 + (2/r^2)(\partial v/\partial\theta)^2 + (1/r^2)(\partial w/\partial\theta)^2 + (3u/r^2)(\partial v/\theta) + (1/r)(\partial u/\partial\theta) + (1/r)(\partial u/\partial\theta)(\partial v/\partial r) + (u/r)(\partial^2 v/\partial r\partial\theta) + (v/r^2)(\partial^2 v/\partial\theta^2) +) + (1/r)(\partial w/\partial\theta)(\partial v/\partial z) + (w/r)(\partial^2 v/\partial\theta\partial z) + (u/r)(\partial u/\partial z) + u^2/r^2$,

$e_{zz} = \partial^2 w/\partial t\partial z + (\partial u/\partial z)^2 + (\partial v/\partial z)^2 + 2(\partial w/\partial z)^2 + (\partial u/\partial z)(\partial w/\partial r) + u(\partial^2 w/\partial r\partial z) + (1/r^2)(\partial v/\partial z)(\partial w/\partial\theta) + (v/r)(\partial^2 w/\partial\theta\partial z) + w(\partial^2 w/\partial z^2)$,

$e_{r\theta} = (1/2)[(\partial/\partial t)\{1/r)(\partial u/\partial\theta) + \partial v/\partial r - v/r\} + (3/r)(\partial u/\partial r)(\partial u/\partial\theta) + (3/r)(\partial v/\partial r)(\partial v/\partial\theta) + (1/r^2)(\partial u/\partial\theta)(\partial y/\partial\theta) + (1/r)(\partial u/\partial z)(\partial w/\partial\theta) + (2/r)(\partial w/\partial r)(\partial w/\partial\theta) + (\partial u/\partial r)(\partial v/\partial r) + (\partial v/\partial z)(\partial w/\partial r) + (u/r)(\partial^2 u/\partial r\partial\theta) + u(\partial^2 v/\partial r^2) + (v/r^2)(\partial^2 u/\partial\theta^2) + (v/r)(\partial^2 v/\partial r\partial\theta) + (w/r)(\partial^2 u/\partial\theta\partial z) + w(\partial^2 u/\partial r\partial z) - (4v/r^2)(\partial v/\partial\theta) - (v/r)(\partial u/\partial r) + (2u/r)(\partial v/\partial r) - (w/r)(\partial v/\partial z) - (2uv/r^2)]$,

$e_{\theta z} = (1/2)[(\partial/\partial t)\{(\partial v/\partial z) + (1/r)(\partial w/\partial\theta)\} + (2/r)(\partial u/\partial\theta)(\partial u/\partial z) + (3/r)(\partial v/\partial\theta)(\partial v/\partial z) + (3/r)(\partial w/\partial\theta)(\partial w/\partial z) + (\partial u/\partial z)(\partial v/\partial r) + (\partial v/\partial z)(\partial w/\partial z) + (1/r)(\partial u/\partial\theta)(\partial w/\partial r) - (v/r)(\partial u/\partial z) + (3u/r)(\partial v/\partial z) + (1/r^2)(\partial v/\partial\theta) + u(\partial^2 v/\partial r\partial z) + (v/r)(\partial^2 v/\partial\theta\partial z) + (u/r)(\partial^2 w/\partial r\partial\theta) + (v/r^2)(\partial^2 w/\partial\theta^2) + w(\partial^2 v/\partial z^2) + w(w/r)(\partial^2 w/\partial\theta\partial z)]$,

$e_{zr} = (1/2)[(\partial/\partial t)(\partial u/\partial z + \partial w/\partial r) + 3(\partial u/\partial r)(\partial u/\partial z) + 2(\partial v/\partial r)(\partial v/\partial z) + 3)(\partial w/\partial r)(\partial w/\partial z) + (1/r)(\partial u/\partial\theta)(\partial v/\partial z) - (2v/r)(\partial v/\partial z) + (\partial u/\partial z)(\partial w/\partial z) + (\partial u/\partial r)(\partial w/\partial r) + (1/r)(\partial v/\partial r)(\partial w/\partial\theta) - (v/r^2)(\partial w/\partial\theta) + u(\partial^2 u/$

$\partial r\partial z)+u(\partial^2 w/\partial r^2)+(v/r)(\partial^2 u/\partial\theta\partial z)+(v/r)\ (\partial^2 w/\partial\theta\partial z)+w\{(\partial^2 u/\partial z^2)+\partial^2 w/\partial r\partial z)\}],$ (B.8)

and

$c_{rr}=(\partial u/\partial r)^2+(1/4)[1/r^2)(\partial u/\partial\theta)^2+(\partial u/\partial z)^2+(\partial v/\partial r)^2+(\partial w/\partial r)^2+v^2r^2+2\{(1/r)+(\partial u/\partial\theta)\ (\partial v/\partial r)+\ (\partial u/\partial z)\ (\partial w/\partial r)-(v/r^2)(\partial u/\partial\theta)-(v/r)(\partial v/\partial r)\}],$

$c_{\theta\theta}=(1/r^2)(\partial v/\partial\theta)^2+(2u/r^2)(\partial v/\partial\theta)^2+u^2/r^2+(1/4)[1/r^2)+(\partial u/\partial\theta)^2+(\partial v/\partial r)^2+(\partial v/\partial z)^2+2\{(1/r^2)+(\partial w/\partial\theta)^2+v^2/r^2+2\{(1/r)(\partial u/\partial\theta)(\partial v/\partial r)\quad -(v/r)(\partial v/\partial r)+(1/r)\ (\partial v/\partial z)\ (\partial w/\partial\theta)\}],$

$c_{zz}=(\partial w/\partial z)^2+(1/4)(\partial u/\partial z)^2+(\partial v/\partial z)^2+(\partial w/\partial r)^2+(1/r^2)(\partial w/\partial\theta)^2+2\{(\partial u/\partial z)(\partial w/\partial r)+(1/r)(\partial v/\partial z)(\partial w/\partial\theta)\}],$

$c_{r\theta}=(1/2)[(1+r)(\partial u/\partial r)(\partial u/\partial\theta)+(\partial u/\partial r)(\partial v/\partial r)+(1+r^2)(\partial u/\partial\theta)(1+r)(\partial v/\partial r)(\partial v/\partial\theta)-(v/r)(\partial u/\partial r)+(u/r^2)(\partial u/\partial\theta)+(u/r)(\partial v/\partial r)-(v/r^2)\quad(\partial v/\partial\theta)-(uv/r^2)](1+4)[\ (\partial u/\partial z)\ (\partial v/\partial z)+(1/r)\ (\partial u/\partial z)(\partial w/\partial\theta)+(\partial v/\partial z)(\partial w/\partial r)+(1+r)(\partial w/\partial r)(\partial w/\partial\theta)],$

$c_{\theta z}=(1/2)[(1+r)(\partial v/\partial\theta)(\partial v/\partial z)+(1/r^2)(\partial v/\partial\theta)(\partial w/\partial\theta)+(\partial w/\partial\theta)+(\partial v/\partial z)\ (\partial w/\partial z)+(1/r)(\partial w/\partial z)(\partial w/\partial\theta)+(u/r)(\partial v/\partial z)+(u/r^2)(\partial w/\partial\theta)]+(1+4)(1/r)\ (\partial u/\partial\theta)(\partial u/\partial z)+(1/r)(\partial u/\partial\theta)(\partial w/\partial r)+(\partial v/\partial r)(\partial u/\partial z)+(\partial v/\partial r)(\partial w/\partial r)-(v/r)(\partial u/\partial z)-(v/r)(\partial w/\partial r)],$

$c_{zr}=(1/2)(\partial u/\partial r)(\partial u/\partial z)+(\partial u/\partial r)(\partial w/\partial r)+(\partial u/\partial z)(\partial w/\partial z)\quad +(\partial w\partial z)+(1/r)(\partial w/\partial z)(\partial w/\partial r)+(\partial w/\partial z)+(1+4)[(1/r)(\partial u/\partial\theta)\quad(\partial v\partial z)+(1/r^2)(\partial u/\partial\theta)(\partial w/\partial\theta)+(\partial v/\partial r)(\partial v/\partial z)+(1+r)(\partial v/\partial r)(\partial w/\partial\theta)-(v/r)(\partial v/\partial z)-(v/r^2)(\partial w/\partial\theta)].$ (B.9)

The energy equation (1.10) transform to

$\rho c_v(DT/Dt) = k\nabla^2 T + \phi,$ (B.10)

$$\nabla^2 = \partial^2/\partial r^2 + (1/r)(\partial/\partial r) + (1/r^1)(\partial^2/\partial\theta^2) + (\partial^2/\partial z^2),$$

And

$$\phi = \tilde{\tau}_{rr} d_{rr} + \tilde{\tau}_{\theta\theta} d_{\theta\theta} + \tilde{\tau}_{zz} d_{zz} + 2(\tilde{\tau}_{r\theta} d_{r\theta} + \tilde{\tau}_{rz} d_{rz} + \tilde{\tau}_{\theta z} d_{\theta z}),$$

(B.11)

$\tilde{\tau}_{rr}$ etc. are the deviatoric tensors.

Chapter No. 2

THE FLOW OF A SECOND-ORDER FLUID BETWEEN TWO INFINITE DISCS IN THE PRESENCE OF TRANSVERSE MAGNETIC FIELD

II.1 INTRODUCTION

The phenomenon arising out of the flow between two discs has important applications in chemical and mechanical engineering as its generalization could be helpful in the study of heat transfer for analysis of air cooling of turbine discs and the determination of oil film temperature of pedestal bearing with centre feeding of lubricant. Several authors have discussed the steady flow of an incompressible viscous fluid between two infinite rotating discs theoretically as well as experimentally. Sharma & Gupta [58)] have considered a general case of the flow of a Newtonian second-order fluid between two infinite torsionally oscillating discs Rajgopal[59)] studied the flow of a second-order fluid between rotating parallel plates. Sharma & Singh[60)] have considered the presence of transverse magnetic field. B. B. Singh and Anil Kumar[62)] have considered the flow of a second-order fluid due to the rotation of an infinite porous disc near a stationary parallel porous disc.

In our present problem, we here study the flow pattern of an incompressible second-order fluid between two parallel infinite discs in the presence of transverse magnetic field when one is rotating (called rotor) and other is at rest (called stator). A uniform injection is applied to the stator forming the subject matter of the paper. The Rotor coincides with the plane $z = 0$ and the stator coincides with the plane $z = d$. Here the dimensionless parameters $\tau_1(\mu_2/pd^2)$, $\tau_2(\mu_2/pd^2)$ govern the effects of elastic-viscosity and cross–viscosity, while the effect of the injection are governed by a non-dimensional parameter k (=$w_0/2d\Omega$) where w_0 is the uniform suction velocity (negative for injection).

II.2 FORMULATION OF THE PROBLEM

The second-order fluid characterized by the equation (1.4) is confined between two infinite discs. The disc coinciding with the plane z = 0 rotates with a uniform angular velocity Ω about z-axis, while the other (assumed porous) coincides with the plane z=d and is at rest. A uniform injection - w_0 (w_0 being positive in z-increasing direction) is applied normal to the stationary disc. The space between these two discs is occupied by homogeneous, electrically conducting, incompressible second-order fluid. Let us suppose that u, v, w are the velocities in r, θ and z directions respectively. Let us suppose that axes of the discs coincide with z-axis and the origin of the cylindrical co-ordinates is taken at the centre of the rotator disc. Under transverse magnetic field of constant intensity B, equation of motion, leaving induced magnetic fields, for steady flow are

$$p\{u(\partial u/\partial r)+w(\partial u/\partial z)-v^2/r\}= \partial\tau_{rr}/\partial r+\partial\tau_{rz}/\partial z+(1/r)(\tau_{rr}-\tau_{\theta\theta})-\sigma B^2 u/p,$$

(2.1)

$$p\{u(\partial v/\partial r)+w(\partial v/\partial z)-uv/r\}=\partial\tau_{r\theta}/\partial r+\partial\tau_{\theta z}/\partial z+2\tau_{r\theta}/r-\sigma B^2 v/p,$$

(2.2)

$$p\{u(\partial w/\partial r)+w(\partial w/\partial z)\}=\partial\tau_{rz}/\partial r+\partial\tau_{zz}/\partial z+\tau_{rz}/r$$

(2.3)

Assuming u, v, w are the radial, transverse and axial components of velocity respectively. ρ and σ are respectively density and the conductivity of the fluid considered. The relevant boundary conditions of the problem are:

z = 0:	u = 0,	v = rΩ,	w = 0,
z = d:	u = 0,	v = 0,	w = -w--,

The velocity components for axi-symmetric flow compatible with continuity criterion (B.1) can be taken as (cf. Von-Karman[63)])

$u = r\Omega F'(\zeta),$

$v = r\Omega G'(\zeta),$

$w = -2d\Omega F'(\zeta),$

(2.5)

and

$P = \Omega\mu_1[-p_1(\zeta)+R\,\xi^2(2\tau_1+\tau_2)(F''^2+G'^2)+\lambda\xi^{2\}}$

Where ζ=z/d, ξ= r/d, $R=\Omega pd^2/\mu_1$, $\tau_1 = \mu_2/pd^2$ (elastic0-viscosity), $\tau_1 = \mu_2/pd^2$ (cross-viscosity). Here the primes denote derivatives w.r.t ζ and λ is and arbitrary constant to be determined from the boundary conditions (2.4) become.

$\zeta = 0$: F = 0, G = 1, F' = 0,

$\zeta = 1$: F = k, G = 0, F' = 0, (2.7)

where $K = w_0/2d\Omega$ is a dimensionless parameter representing the injection of the stator.

Following sets of equations are obtained after substituting (2.5) & (2.6) into (1.4), (2.1), (2.2), (2.3)

$R(F'^2-2FF''-G^2) = F'''-2R\tau_1(F''^2+2G'^2+FF^{iv}) - R\tau_2(F''^2+3G'^2+2F'F''') + m^2F'-2\lambda$ (2.8)

$2R(F'G-2FG') = G''-2R\tau_1(F''G'-FG''') + 2R\tau_2(F''G'-F'G'') - m^2G$ (2.9)

$4RFF' = p_1' - 2F'' + 4R\tau 1\ (11\ F'F'' + FF''') + 28R\tau_2 F'F''$

(2.10)

where $m^2 = B^2 \sigma d^2/\mu_1$ is the dimensionless constant.

II.3 SOLUTION OF THE PROBLEM

Assuming the relationship $m^2 = Rm^2_1$, equations (2.8) & (2.9) becomes

$R(F'^2 - 2FF'' - G^2) = F''' - 2R\tau_2(F''^2 + 2G'^2 + FF^{iv}) - R\tau_2(F''^2 + 3G'^2 + F'FF''') + Rm_1^2 F' - 2\lambda$ (2.11)

$2R(F'G - FG') = G'' + 2R\tau_1(F''G' - FG''') + 2R\tau_2(F''G' - F'G') - Rm_1^2 G$

(2.12)

Taking R to be small, we expand F, G and λ in ascending powers of R as follows:

$F = F_0 + Rf_1 + R^2 f_2 + \ldots\ldots\ldots..$

$G = g_0 + Rg_1 + R^2 g_2 + \ldots\ldots\ldots$

$\lambda = \lambda_0 + R\lambda_1 + R^2 \lambda_2 + \ldots\ldots\ldots..$

(2.13)

Substituting (2.13) in (2.11) and (2.12), and on equating the constant terms and the similar powers of R on both the sides, neglecting the powers higher than two, we get the following sets of linear differential equations:

$f_0''' = 2\lambda_0,$

$f_1''' = f_0'^2 g_0^2 - 2f_0 f_0'' + 2\tau_1(f_0''^2 + 2g_0'^2 + f_0 f_0^{iv}) + \tau_2(f_0''^2 + 3g_0'^2 + f_0' f_0''') - m_1^2 f_0' + 2\lambda_1,$

$f_2''' = 2f_0' f_1' - 2f_1 f_0'' - 2f_1 f_0'' - 2g_0 g_1 + 2\tau_1 (2f_0'' f_1'' + 4g_0 g_1 + f_1 f_0^{iv} + f_0 f_1^{iv}) +$
$2\,\tau_2(f_0'' f_1'' + 3g_0' g_1' + f_0' f_0''') - m_1^2 f_1' + 2\lambda_1.$

(2.14)

$g_0'' = 0,$

$g_1'' = 2(f_1'g_0 - f_0 g_0') - 2\tau_1(f_0''g_0, - f_0 g_0''') - 2\tau_2(f_0''g_0' - f_0'g_0'') + m_1^2 g_0',$

$g_2'' = 2(f_1'g_0 + f_0'g_1' - f_1'g_0 + f_0'g_1') - 2\tau_1(f_1''g_0 + f_0''g_1' - f_1 g_0''' - f_0 g_1''') - 2\tau_2(f_1''g_0' -$

$f_1'g_0'' - f_0'g_1'') + m_1^2 g_0'$

(2.15)

The boundary conditions (2.7) become

$f_n'(0) = 0 \ \forall n,$ $g_0'(0) = 1,$ $g_n'(0) = 0$ $n>0,$ $f_n'(0) = 0 \ \forall n,$
$f_n'(1) = 0 \ \forall n,$ $g_n'(1) = 0 \ \forall n,$ $f_0'(1) = k,$ $f_n'(1) = 0$ $n > 0$,
(2.16)

The solutions of the set of equations (2.14) & (2.15), satisfying the boundary conditions (2.16) are given by:

$f_0 = k(3\zeta^2 - 2\zeta^3),$

$f_1 = -(3k^2 3105)(2\zeta^7 - 7\zeta^6 + 18\zeta^3 - 13\zeta^2) - (1/60)(\zeta^5 - 5\zeta^4 + 7\zeta^3 - 3\zeta^2) + \{12(\tau_1 + \tau_2)k^2/5\})(2\zeta^5 - 5\zeta^4 + 4\zeta^3 - 3\zeta^2)$

II.4 RESULTS AND CONCLUSION

The variation of radial velocity for different elastic-viscous parameter τ_1 = -1.3, -2, -2.6; when cross-viscous parameter τ_1 = 10, injection parameter k = 5 Reynolds number R= 0.05, magnetic field m_1 = 5 is shown in fig (1). In this figure, the curve of radial velocity w.r.t ζ is bell shaped with maximum at $\zeta = 0.5$ approximately. It is also evident from this from this figure that the radial velocity decreases with increase in τ_1 from ζ = 0.0-0.28, then it begins increase with increases in τ_1 upto ζ = 0.72 and then decreases with increase in τ_1 from ζ = 0.8-0.95. The value of radial velocity is approximately equal at ζ

=0.28 and ζ – 0.72 for all values of τ_1. The point of maxima is in the middle of the gap length for all values of elastic-viscous parameter τ_1.

The variation of transverse velocity for different elastic-viscous parameter τ_1 = -1.3. -2, -2.6; when cross-visous parameter τ_1 = 10, injection parameter k = 5, Reynolds number R = 0.05, magnetic field m_1 = 5 is shown in fig (2). In this figure, the transverse velocity increases upto ζ = 0.7 and decreases thereafter. It is also evident from this figure that the transverse velocity decreases with increase in τ_1 throughout the gap-length.

The variation of axial velocity for different elastic-visous parameter τ_1 = -1.3, -2, -2.6; when cross-viscous parameter τ_1 = 10, injection parameter k = 5, Reynolds number R = 0.05, magnetic field m_1 = 5 is shown (3). In this figure, the axial velocity increases throughout the gap-length. It is also evident from this figure that the axial velocity decreases with increase in τ_1 in the first half and in increases with increase in τ_1 in the second half of the gap-length.

The variation of radial velocity for different cross-viscous parameter τ_2 = 3, 5, 7; when elastic-viscous parameter τ_1 = -1 injection parameter k = 5, Reynolds number R = 0.05, magnetic field m_1 = 5 is shown in fig (4). In this figure, the curve of radial velocity w.r.t ζ is bell shaped with maximum at ζ = 0.5 approximately. It is also evident from this figure that the radial velocity decreases with an increase in τ_2 from ζ = 0.0-0.28, then it begins increase with an increase in τ_2 upto ζ = 0.72 and then decreases with an increase in τ_2 from ζ = 0.8-0.95. The value of radial velocity is approximately equal at ζ = 0.28 and ζ = 0.72 for all values of τ_2. The point of maxima is in the middle of the gap-length for all values of cross-viscous parameter τ_2.

The variation of transverse velocity for different cross-viscous parameter τ_2 = 3, 5, 7; when elastic-viscous parameter τ_1 = -1, injection parameter k = 5, Reynolds number R = 0.05, magnetic field m_1 = 5 is shown in fig (5) In this figure, the transverse velocity increases upto $\zeta = 0.7$ and decreases thereafter. It is also evident from this figure that the transverse velocity decreases with an increase in τ_2 upto near the lower disc and then increases with an increase in τ_2 from $\zeta = 0.3$ to $\zeta = 1.0$.

The variation of axial velocity for different cross-viscous parameter τ_2 = 3, 5, 7; when elastic-viscous parameter τ_1 = -1, injection parameter k = 5, Reynolds number R = 0.05, magnetic field m_1 = 5 is shown in fig (6) In this figure, the axial velocity increases throughout the gap-length. It is also evident from this figure that the axial velocity decreases with an increase in τ_2 in the first half and it increase in τ_2 in the second half of the gap-length.

The variation of radial velocity for different injection parameter k = 3, 5, 6; when elastico-viscous parameter τ_1 = -0.4 cross-viscous parameter τ_2 = 2, Reynolds number R = 0.05, magnetic field m_1 = 5 is shown in fig (7) In this figure, the curve of radial velocity w.r.t. ζ is parabolic with vertex upward and the radial velocity increases in the first half and then decreases in the second half of the gap-length. It is also evident from this figure that the radial velocity increases with an increases in injection parameter throughout the gap-length. The point of maxima is in the middle of the gap length for all values of injection parameter k.

The variation of transverse velocity for different injection parameter k = 3, 5, 6; when elastico-viscous parameter τ_1 = -0.4 cross-viscous parameter τ_2 = 2, Reynolds number R = 0.05, magnetic field m_1 = 5 is shown in fig (8) In this figure, at k= 5, 6 the transverse velocity decreases first then increases and start decreasing rapidly thereafter. At k = 3, The transverse velocity decreases throughout the gap-length. It is also evident from this figure that the transverse velocity increases with an increases in injection parameter k throughout the gap-length.

The variation of axial velocity for different injection parameter k = 3, 5, 6; when elastico-viscous parameter τ_1 = -0.4 cross-viscous parameter τ_2 = 2, Reynolds number R = 0.05, magnetic field m_1 = 5 is shown in fig (9) In this figure, the axial velocity increases throughout the gap-length. It is also evident that it also increases with an increases in injection parameter k throughout the gap-length.

The variation of radial velocity for different in Reynolds number R = 0.01, 0.06, 0.09; when elastico-viscous parameter τ_1 = -0.4 cross-viscous parameter τ_2 = 2, injection parameter k = 4 magnetic field m_1 = 5 is shown in fig (10) In this figure, the curve of radial velocity w.r.t. ζ is parabolic with vertex upward and the radial velocity increases in the first half and then decreases in the second half of the gap-length. It is also evident from this figure that the radial velocity decreases with an increases in Reynolds number R ζ = 0.0-0.28, then it begins increases with an increase in Reynolds number R upto ζ = 0.72 and then decreases with an increase in Reynolds number R from ζ = 0.8-0.95. The value of radial velocity is approximately equal at ζ = 0.28 and ζ = 0.72 for all values of Reynolds number R. The point is in the middle of the gap-length for all values of Reynolds number R.

The variation of transverse velocity for different in Reynolds number R = 0.01, 0.06, 0.09; when elastico-viscous parameter τ_1 = -0.4 cross-viscous parameter τ_2 = 2, injection parameter k = 4 magnetic field m_1 = 5 is shown in fig (11) In this figure, at R = 0.01, the transverse velocity decreases thoughout the gap length. At R= 0.06, 0.09, the transverse velocity decreases near the lower disc then increases and decreases rapidly thereafter. It is also evident from this figure that the transverse velocity decreases with an increase R up to ζ = 0.17 and then increases with an increase in R.

The variation of axial velocity for different in Reynolds number R = 0.01, 0.06, 0.09; when elastico-viscous parameter τ_1 = -0.4 cross-viscous parameter τ_2 = 2, injection parameter k = 4 magnetic field m_1 = 5 is shown in fig (12) In this figure, the axial velocity increases throughout the gap-length. It is also evident from this figure that the axial velocity decreases in the first half and it increases in the second half of the gap-length.

The variation of radial velocity for different magnetic field m_1 = 1, 10, 15; when elastico-viscous parameter τ_1 = -0.4 cross-viscous parameter τ_2 = 2, injection parameter k = 4, Reynolds number R = 0.05 is shown in fig (13). In this figure, the curve of radial velocity w.r.t. ζ is parabolic with vertex upward and the radial velocity increases in the first half and then decreases in the second half of the gap-length. It is also evident from this figure that the radial velocity decreases with an increases in magnetic field m_1 upto ζ = 0.0-0.28, then it begins increases with an increase in magnetic field m_1 upto ζ = 0.72 and then decreases with increase in magmetic field m_1 from ζ = 0.8-0.95. The value of radial velocity is approximately equal at ζ = 0.28 and ζ = 0.72 for all values of magnetic field m_1. The point of maxima is in the middle of the gap-length for all values of magnetic field m_1.

The variation of transversel velocity for different magnetic field m_1 = 1, 10, 15; when elastico-viscous parameter τ_1 = -0.4 cross-viscous parameter τ_2 = 2, injection parameter k = 4, Reynolds number R = 0.05 is shown in fig (14). In this figure, the transverse velocity decrease first then increases and decreases thereafter. It is also evident tranverse velocity decreases with increase in magnetic field m_1.

The variation of axial velocity for different in magnetic field m_1 = 1, 10, 15; when elastico-viscous parameter τ_1 = -0.4 cross-viscous parameter τ_2 = 2, injection parameter k = 4 Reynolds number R = 0.05 is shown in fig (15) In this figure, the axial velocity increases throughout the gap-length. It is also evident from this figure that the axial velocity decreases with increase in magnetic field m_1 in the second half of the gap-length.

The transverse shearing stress on the lower and upper disc is

$$(\tau_{\theta z})_{z=z0} = -1+R\{k-2k(\tau_1+\tau_2)-m_1^2/3\} +R^2[-761k^2/3150-3/700 +24k^2 (\tau_1+\tau_2)/5+m_1^2 k/10\} (1/140)+k^2(\tau_1+\tau_2)/5+m_1^2k/15-2\tau_1\{76361k^2/47775+\{24 k^2(\tau_1+\tau_2)/5+m_1^2k/10\} (9/4)+22k^2(\tau_1+\tau_2)/5+13m_1^2k/30\}-2\tau_2 \{2k^2(\tau_1+\tau_2)-367k^2/2940 +22m_1^2k/39\}-551m_1^2 k(\tau_1+\tau_2)/30+m_1^4/45],$$

And

$$(\tau_{\theta z})_{z=z0} = -1-27Rk/10-2kR(\tau_1+\tau_2)+m_1^2R/6+R^2[3061k^2 3150+2/1575+ 24k^2(\tau_1+\tau_2) /5+m_1^2k/10\}(5883/420)+k^2(\tau_1+\tau_2)/5-2m_1^2k/15+2\tau_1\{199859k^2/47775+ \{24k^2(\tau_1+\tau_2)/5+m_1^2k/10\}(27/4)+38k^2(\tau_1+\tau_2)/5+17m_1^2k/30\}-2\tau_2\{613k^2/2940+2k^2(\tau_1+\tau_2)-14m_1^2k/39\} +337m_1^2 k/2100+ m_1^2 k(\tau_1+\tau_2)/30-7m_1^4/360].$$

Respectively.

	U			V		
Z	τ_{ij} = -1.3	τ_{ij} = -2	τ_{ij} = -2.6	τ_{ij} = -1.3	τ_{ij} = -2	τ_{ij} = -2.6
0	0	0	0	1	1	1
0.1	0.061084	0.272359	0.453451	0.724947	1.802216	2.580493
0.2	3.107262	3.244516	3.362162	1.648343	3.664370	5.108447
0.3	6.886935	6.842518	6.804446	3.621123	6.423730	8.412976
0.4	9.782090	9.576825	9.400883	6.245975	9.657036	12.056344
0.5	10.827963	10.560393	10.331047	8.947225	12.749060	15.400839
0.6	9.720286	9.514972	9.338989	11.042045	14.962566	17.677098
0.7	6.808657	6.764160	6.726020	11.812569	15.509238	15.509238
0.8	3.075155	3.212326	3.329901	10.578534	13.621144	15.705835
0.9	0.097916	0.309137	0.490184	6.770037	8.622317	9.003630
1.0	0	0	0	0	0	0
	Table (1): response of radial velocity with ζ at different elastic-viscous parameter.			Table (2): response of transverse velocity with ζ at different elastic-viscous parameter.		

	W			U		
Z	$\tau_1 = -1.3$	$\tau_1 = -2$	$\tau_1 = -2.6$	$\tau_2 = 3$	$\tau_2 = 5$	$\tau_2 = 7$
0	0	0	0	1	1	1
0.1	-0.032823	-0.019016	-0.007181	2.083284	1.479642	0.0876000
0.2	0.111048	0.143846	0.171958	4.420979	4.028825	3.636670
0.3	0.612374	0.650098	0.682434	6.461800	6.588706	6.715612
0.4	1.458224	1.482888	1.504029	7.817407	8.403880	8.990352
0.5	2.506414	2.506427	2.506439	8.266933	9.031419	9.795906
0.6	3.551303	3.526663	3.505544	7.755142	8.341752	8.928362
0.7	4.389634	4.351928	4.319609	6.382758	6.509822	6.637026
0.8	4.884974	4.852187	4.824083	4.388078	3.996161	3.604243
0.9	5.029267	5.015462	5.003630	2.119605	1.516116	0.912626
1.0	5	5	5	0	0	0

Table (3): response of axial velocity with ζ at different elastic-viscous parameter.

Table (4): response of radial velocity with ζ at different elastic-viscous parameter.

	V			W		
Z	$\tau_2 = 3$	$\tau_2 = 5$	$\tau_2 = 7$	$\tau_2 = 3$	$\tau_2 = 5$	$\tau_2 = 7$
0	1	1	1	0	0	0
0.1	1.037118	0.999511	0.802352	0.099332	0.059883	0.020434
0.2	1.206587	1.340820	1.246189	0.424966	0.331259	0.237552
0.3	1.479395	1.965678	2.241624	0.973451	0.865667	0.757883
0.4	1.800817	2.750108	3.575751	1.694298	1.623828	1.553358
0.5	2.096254	3.521447	4.946640	2.506544	2.506505	2.506466
0.6	2.277552	4.076089	5.998275	3.315464	3.385865	3.456264
0.7	2.277552	4.197728	6.356143	4.028732	4.136464	4.244196
0.8	1.917407	3.375869	5.663195	4.571151	4.664830	4.758508
0.9	1.192468	3.615884	3.615884	4.897138	4.936579	4.976021
1.0	0	0	0	5	5	5

Table (5): response of transverse velocity with ζ at different cross-viscous parameter.

Table (6): response of axial velocity with ζ at different cross-viscous parameter.

V				W		
Z	k = 3	k = 5	k = 6	k = 3	k = 5	k = 6
0	0	0	0	1	1	1
0.1	1.441270	2.204012	2.517366	0.841930	0.849284	0.860335
0.2	2.777479	4.499410	5.309626	0.738775	0.795300	0.842035
0.3	3.842519	6.436419	7.730562	0.679640	0.819216	0.921788
0.4	4.519018	7.700113	9.329642	0.649546	0.891150	1.060498
0.5	4.739853	8.114036	9.856876	0.630447	0.972879	1.207145
0.6	4.488070	7.637820	9.257867	0.602350	1.020847	1.303076
0.7	3.794879	6.357331	7.656552	0.544515	0.989384	1.286595
0.8	2.735546	4.466462	5.324080	0.436692	0.834033	1.097715
0.9	1.423109	2.240303	2.633390	0.263390	0.260370	0.514913
1.0	0	0	0	0	0	0

Table (7): response of radial velocity with injection.

Table (8): response of transverse velocity with injection.

	W			U		
Z	k = 3	k = 5	k = 6	R = 0.01	R = 0.06	R = 0.09
0	0	0	0	0	0	0
0.1	0.072123	0.107222	0.120388	2.102290	1.773154	1.543200
0.2	2.0.284701	0.443707	0.512431	3.809710	3.606568	3.443329
0.3	0.618525	0.995008	1.169717	5.066212	5.149262	5.160683
0.4	1.040197	1.708392	2.030924	5.832238	6.154601	6.316958
0.5	1.507084	2.506552	2.999698	6.084908	6.483750	6.702499
0.6	1.972360	3.301387	3.964558	5.818043	6.104375	6.273069
0.7	2.389935	4.007186	4.817696	5.041985	5.083501	5.133683
0.8	2.719089	4.552416	5.471381	3.783202	3.573899	3.508065
0.9	2.98575	4.889250	5.870523	2.086663	1.793772	1.693272

1.0	3	5	6	0	0	0

Table (9): response of axial velocity with injection.

Table (10): response of radial velocity different Reynolds number.

	V			W		
Z	R = 0.01	R=0.06	R=0.09	R = 0.01	R = 0.06	R = 0.09
0	1	1	1	0	0	0
0.1	0.887852	0.832946	0.804672	0.108129	0.086429	0.071536
0.2	0.785358	0.763283	0.790949	0.407278	0.356531	0.320374
0.3	0.690367	0.772200	0.925297	0.855022	0.797936	0.753929
0.4	0.600359	0.830712	1.151772	1.404152	1.368351	1.333619
0.5	0.512526	0.902180	1.397593	2.004339	2.006197	1.991444
0.6	0.423872	0.945099	1.579331	2.603798	2.641372	2.646785
0.7	0.331307	0.916075	1.609524	3.150956	3.205593	3.222259
0.8	0.231750	0.772914	1.403543	3.596087	3.641708	3.657231
0.9	0.122233	0.477731	0.886495	3.892889	3.911311	3.917549
1.0	0	0	0	4	4	4

Table (11): response of Transverse velocity with different Reynolds number

Table (12): response of axial velocity with different Reynolds number

	U			V		
Z	$m_1 = 1$	$m_1 = 10$	$m_1 = 15$	$m_1 = 1$	$m_1 = 10$	$m_1 = 15$
0	0	0	0	1	1	1
0.1	1.0872438	1.752882	1.58733	0.910303	0.673590	0.526663
0.2	3.672795	3.592213	3.477583	0.871905	0.489365	0.290839
0.3	5.132934	5.158497	5.191992	0.873426	0.418651	0.228314
0.4	6.066779	6.188827	6.359439	0.896292	0.427239	0.277027
0.5	6.370446	6.528991	6.752092	0.916450	0.477107	0.378110
0.6	6.020021	6.138271	6.304088	0.906273	0.528350	0.477148
0.7	5.066240	5.086140	5.112483	0.836590	0.541239	0.525571
0.8	3.625624	3.540440	3.420000	0.678796	0.478355	0.482126

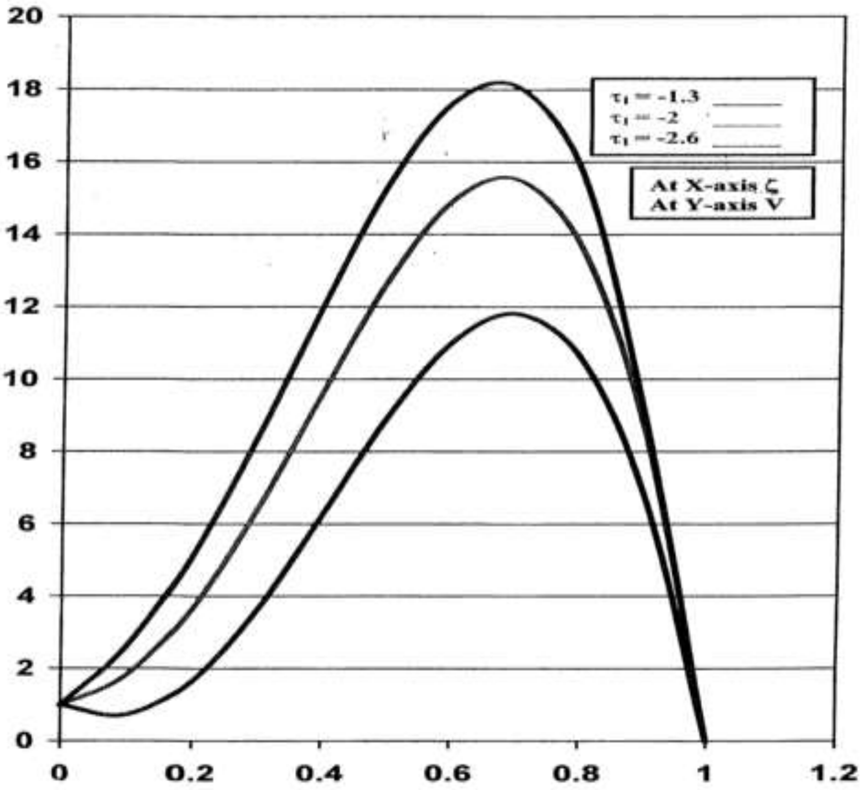
0.9	1.867916	1.746874	1.579445	0.406979	0.306742	0.314353
1.0	0	0	0	0	0	0

Table (13): response of radial velocity with different m_1

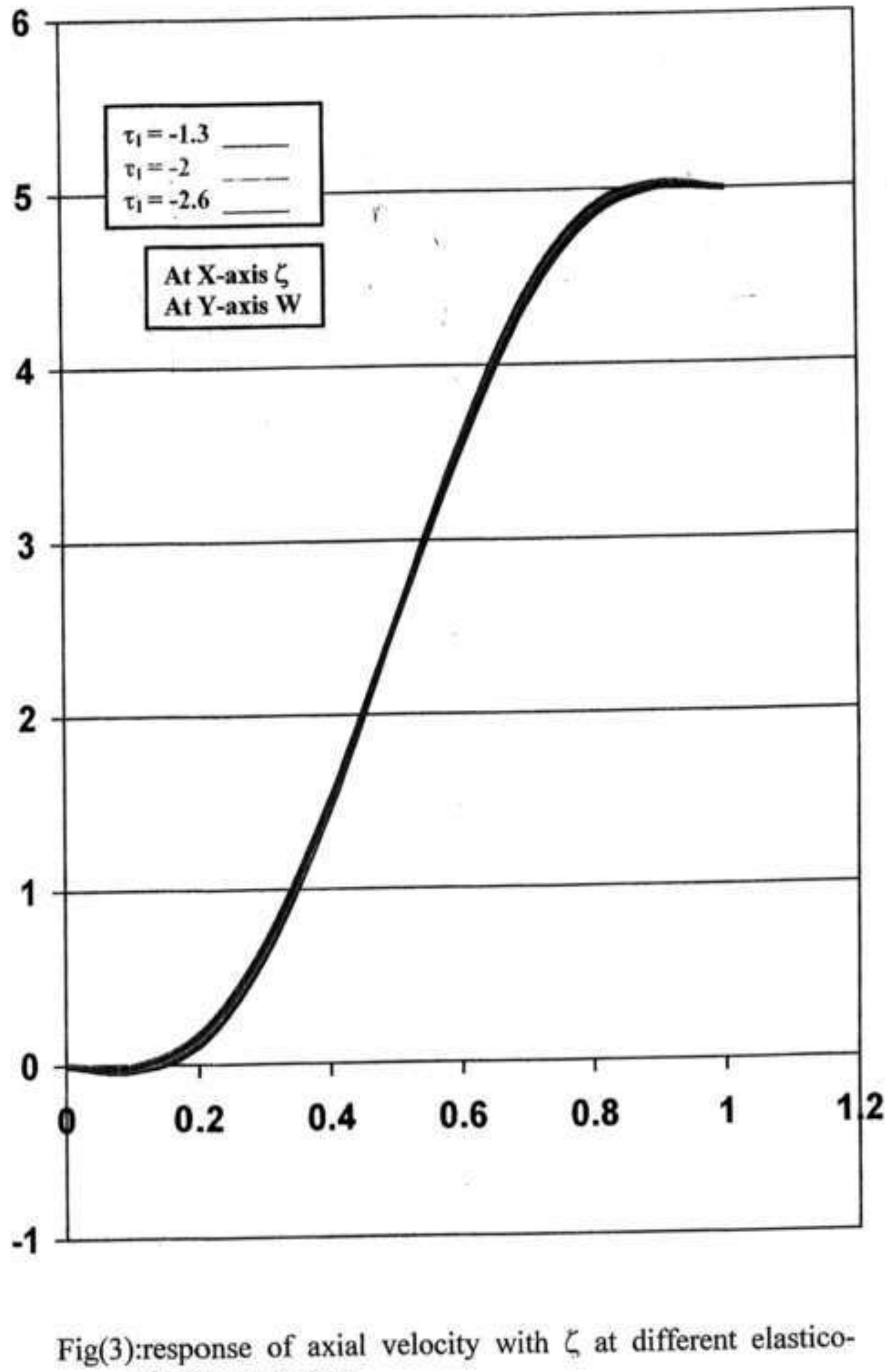
Table (14): response of transverse velocity with different m_1

	W		
Z	$m_1 = 1$	$m_1 = 10$	$m_1 = 15$
0	0	0	0
0.1	0.092891	0.085203	0.074653
0.2	0.371995	0.353370	0.327499
0.3	0.816018	0.794436	0.764186
0.4	1.380947	1.367083	1.347504
0.5	2.008279	2.009068	2.010065
0.6	2.633150	2.648395	2.669717
0.7	3.192098	3.214565	3.245932
0.8	3.630111	3.649083	3.675393
0.9	3.906611	3.914336	3.924931
1.0	4	4	4

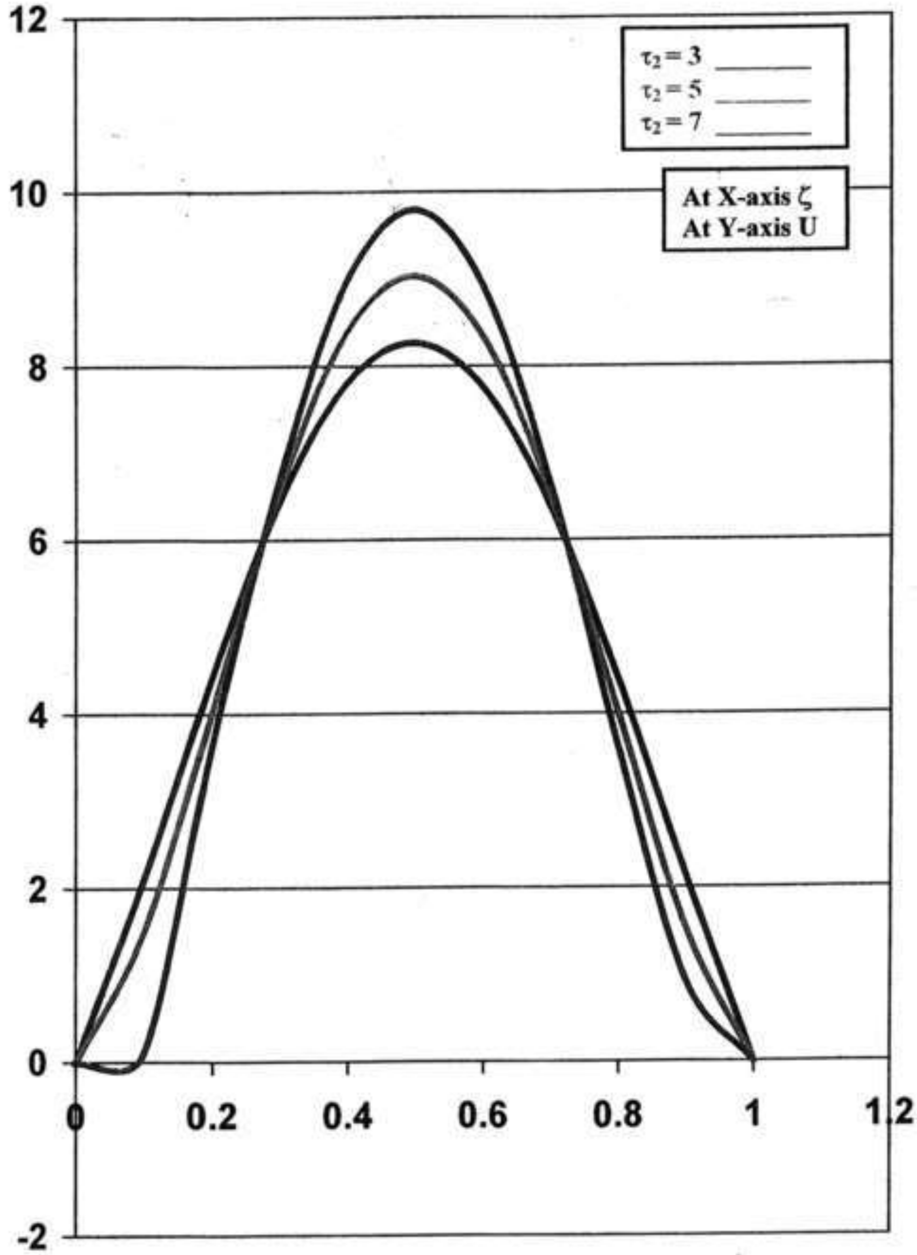
Table(15): Response of axial velocity with different m_1



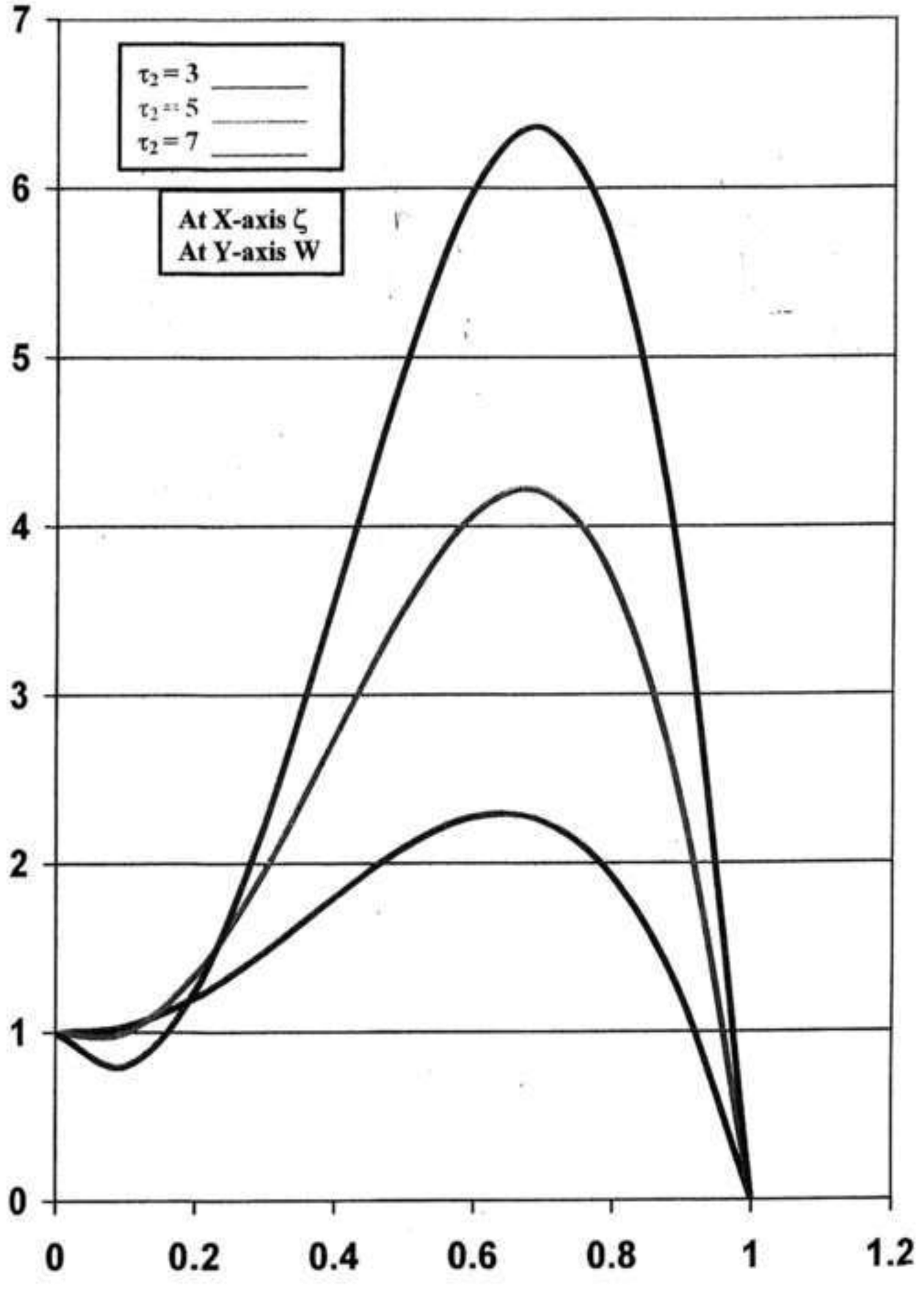
Fig(2):response of transverse velocity with ζ at different elastico-viscous parameter.



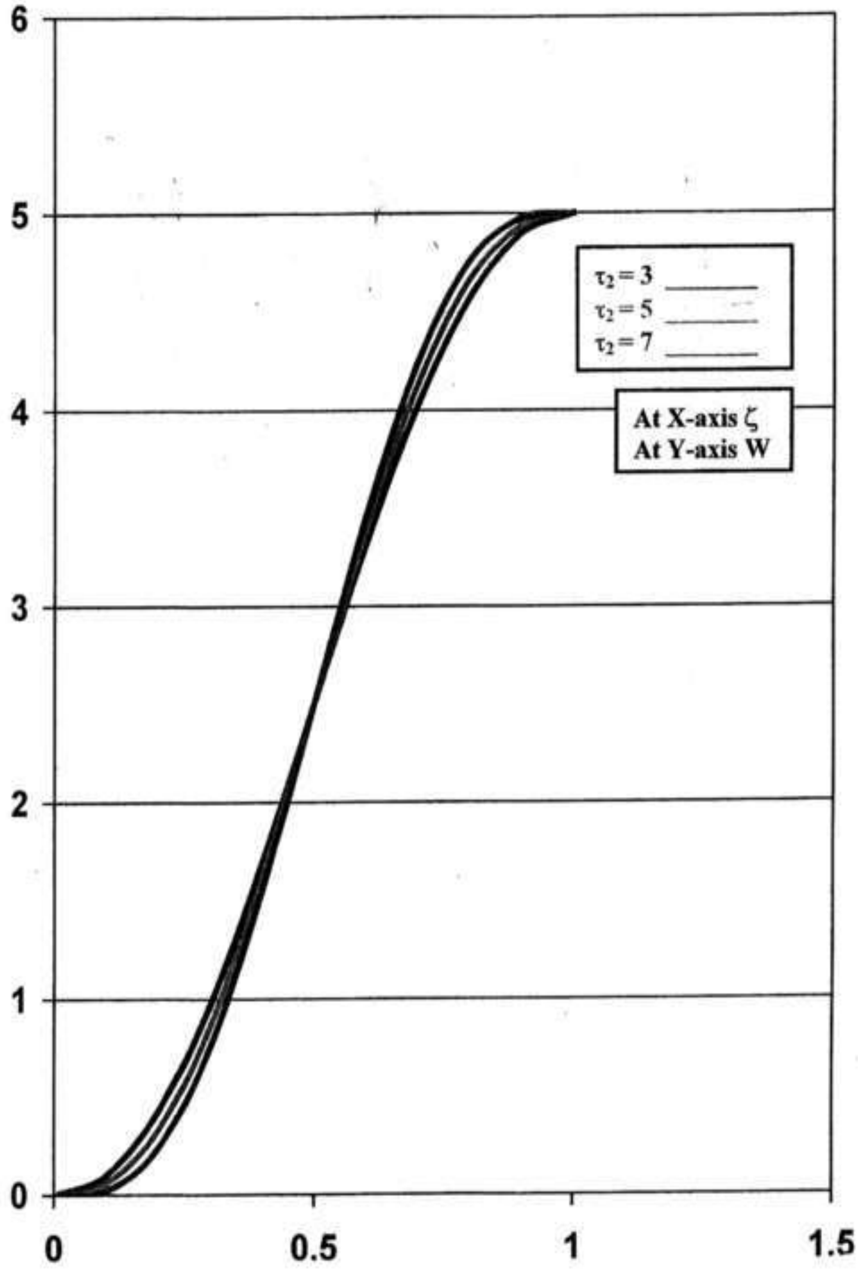
Fig(3):response of axial velocity with ζ at different elastico-viscous parameter.



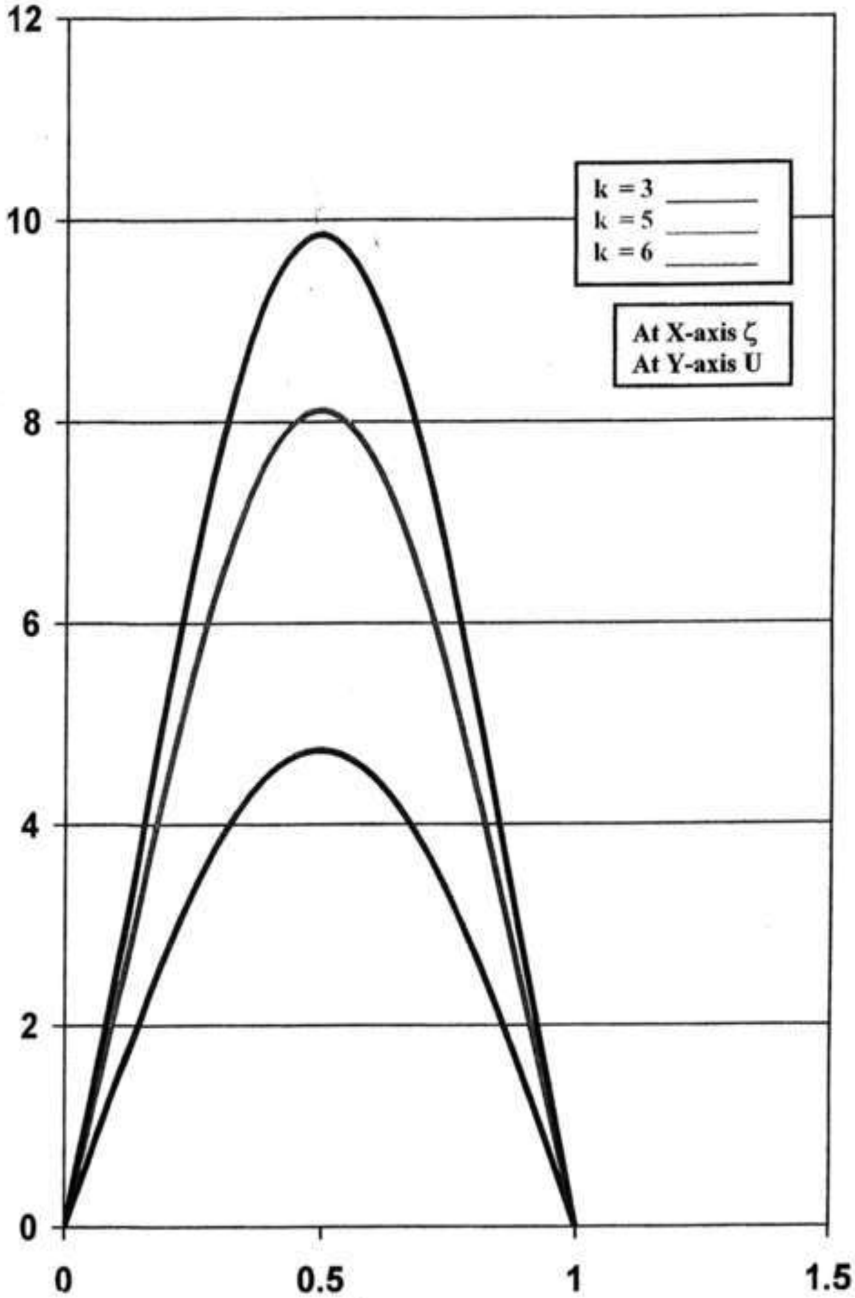
Fig(4):response of radial velocity with ζ at different cross-viscous parameter.



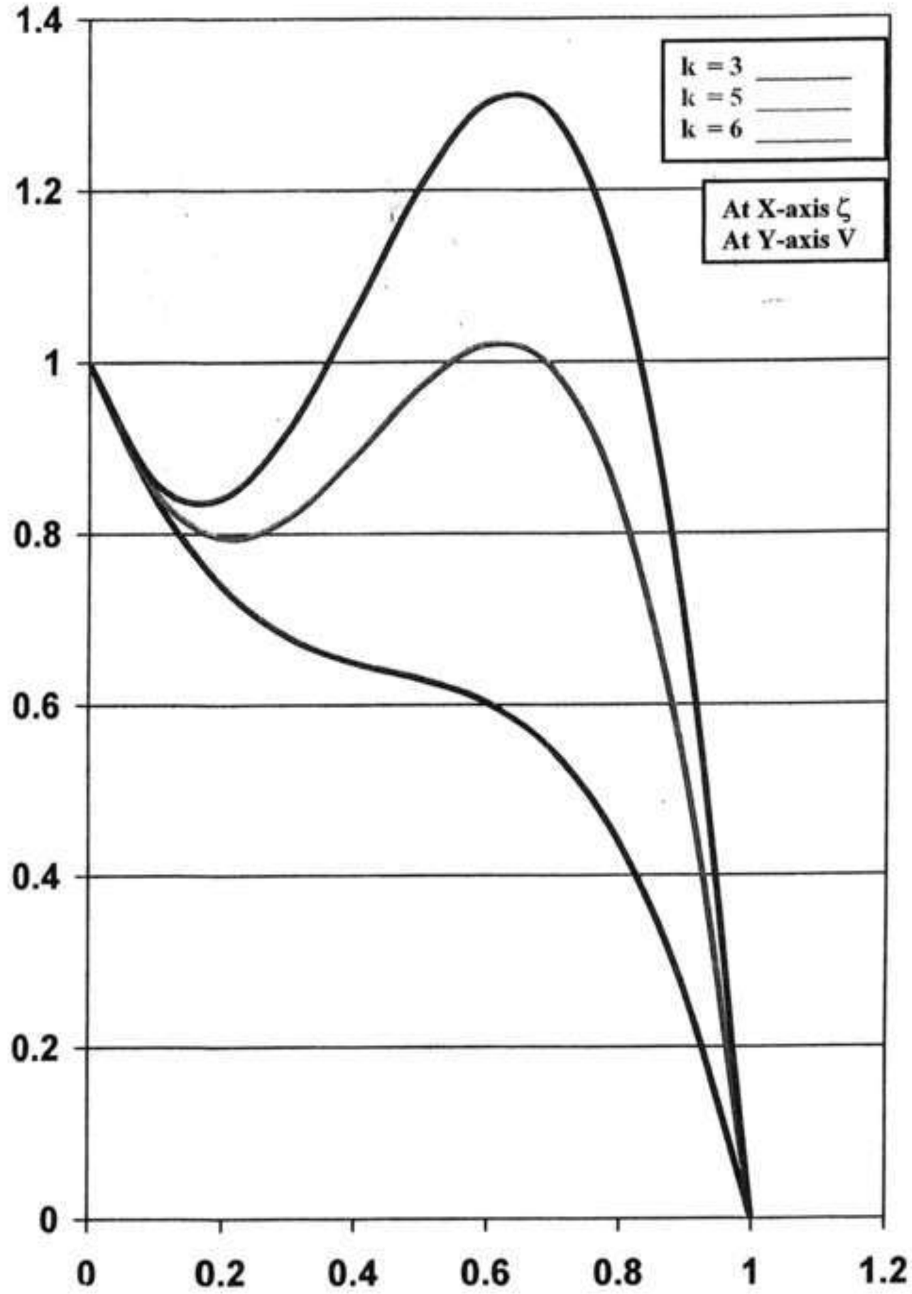
Fig(5):response of transverse velocity with ζ at different cross-viscous parameter.



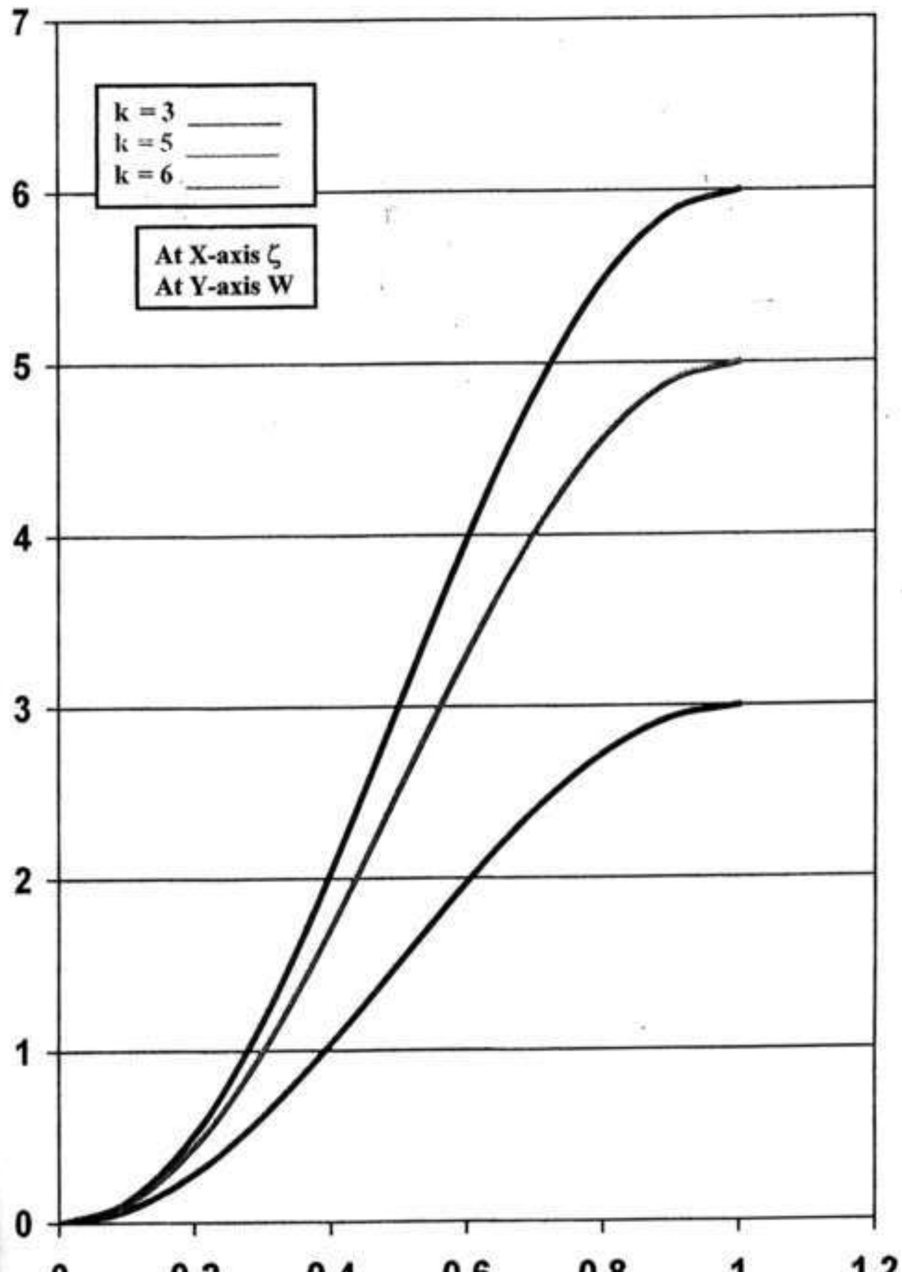
Fig(6):response of axial velocity with ζ at different cross-viscous parameter.



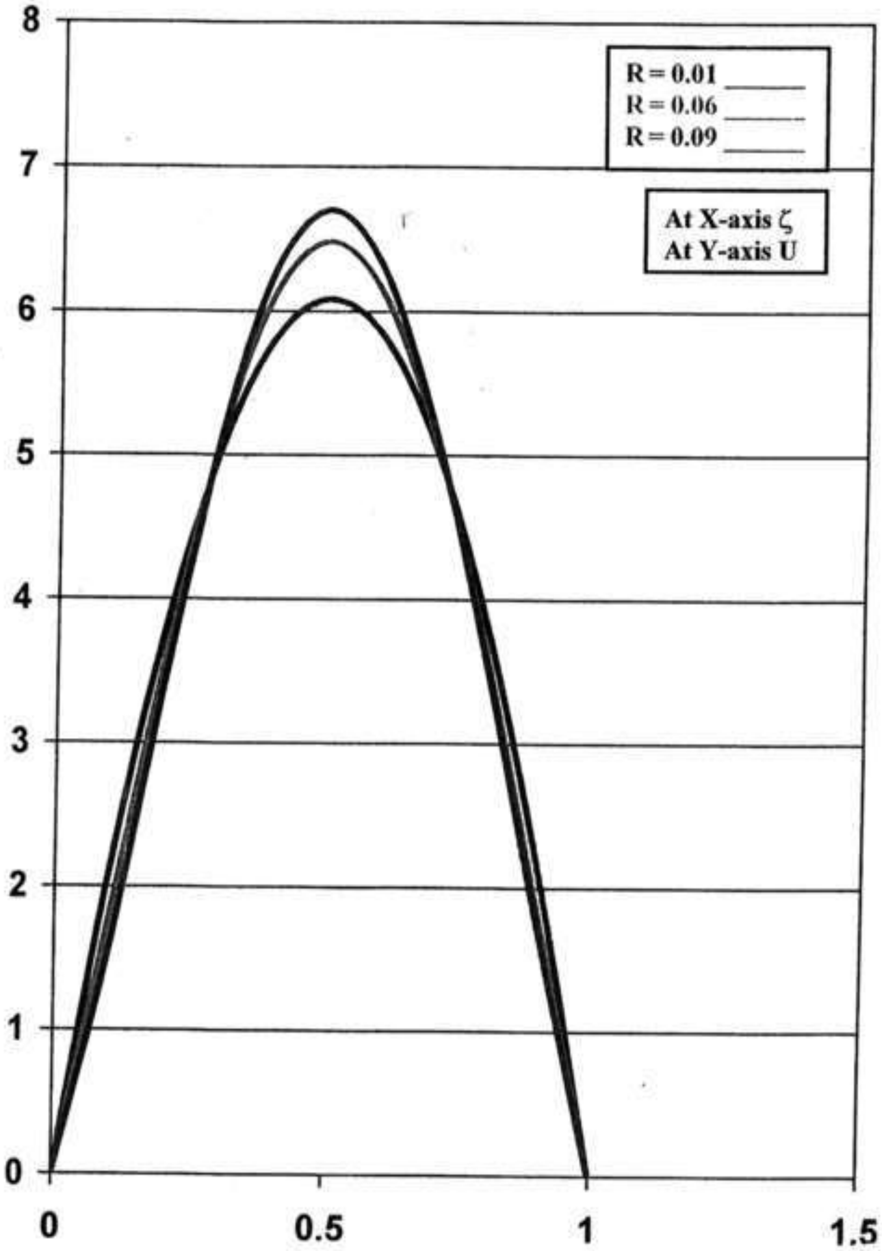
Fig(7):response of radial velocity with injection.



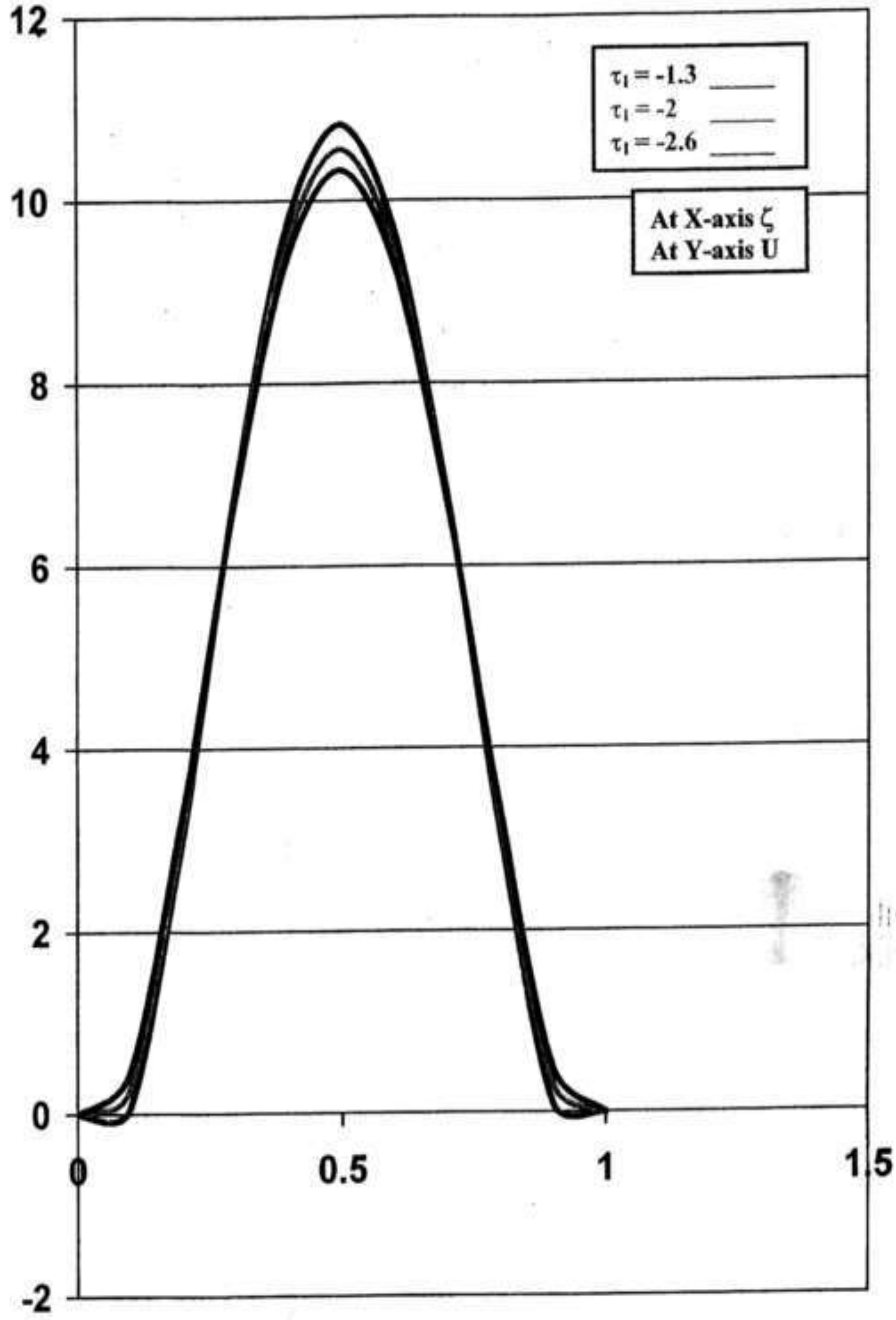
Fig(8):response of transverse velocity with injection.



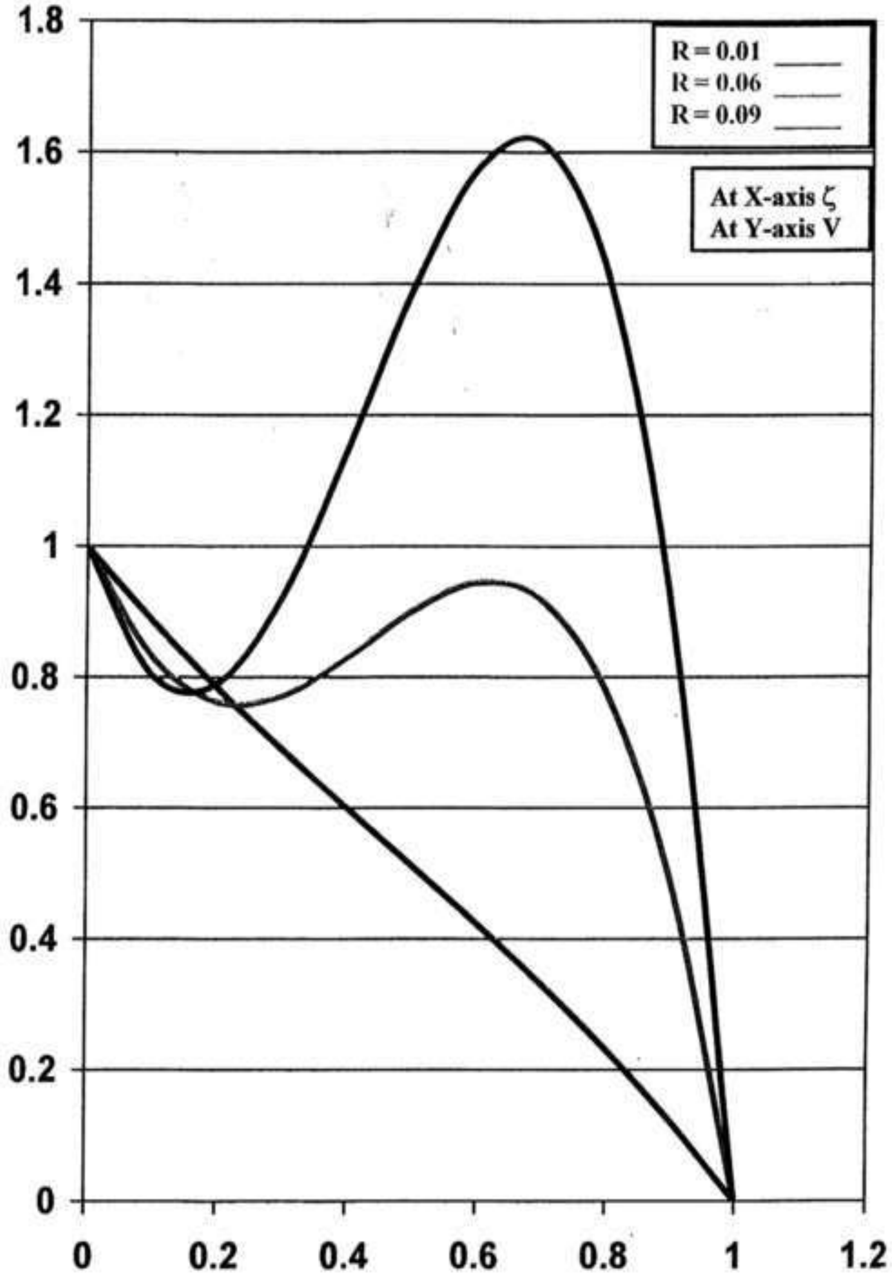
Fig(9):response of axial velocity with injection.



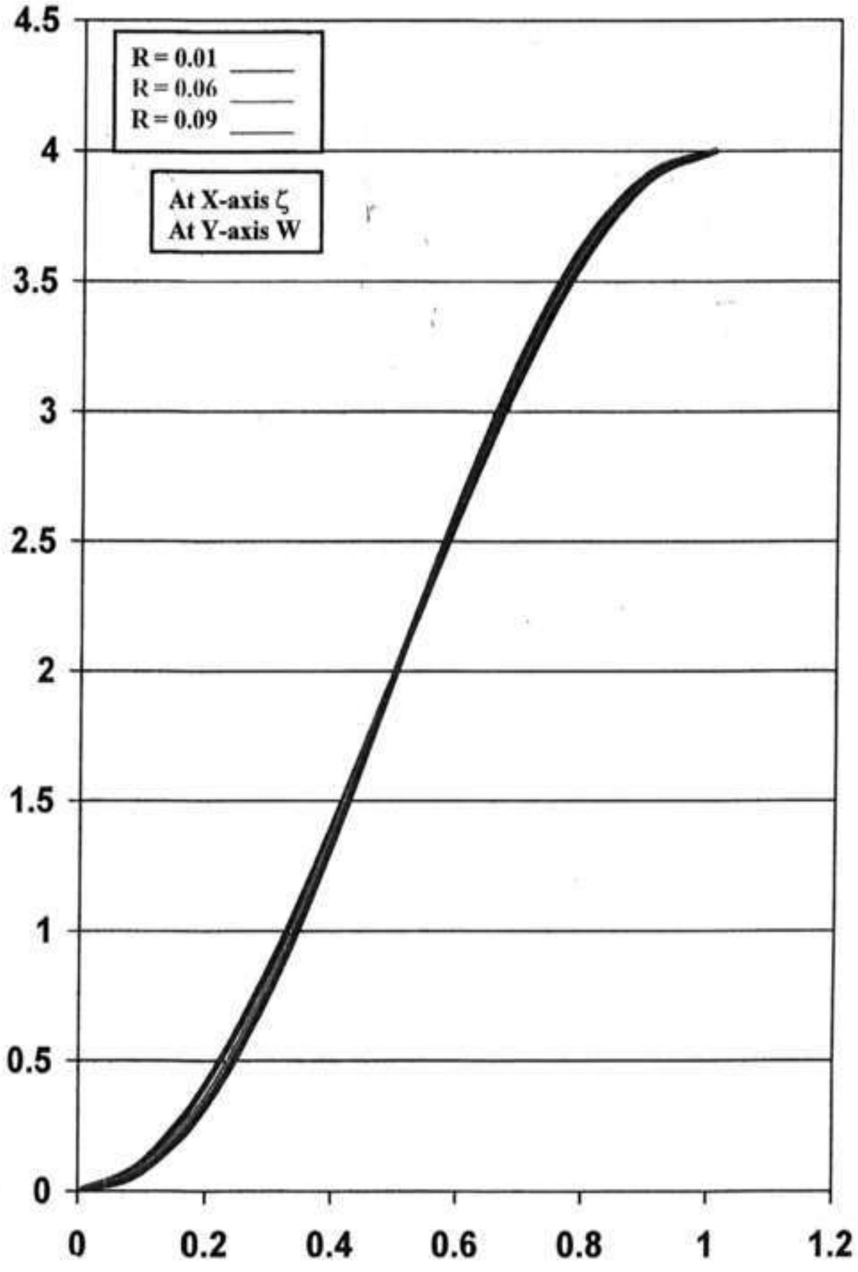
Fig(10):response of radial velocity with different Reynolds number.



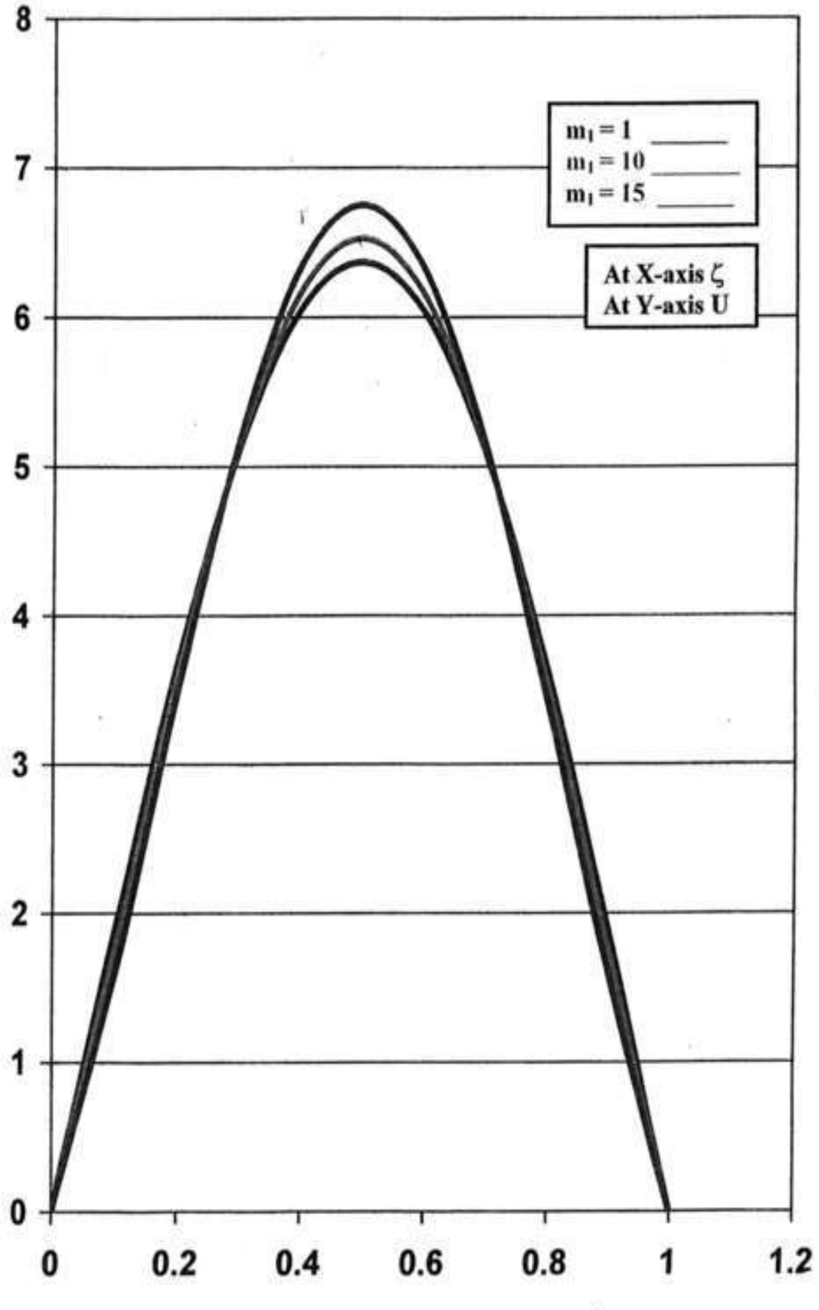
Fig(1): response of radial velocity with ζ at different elastico-viscous parameter.



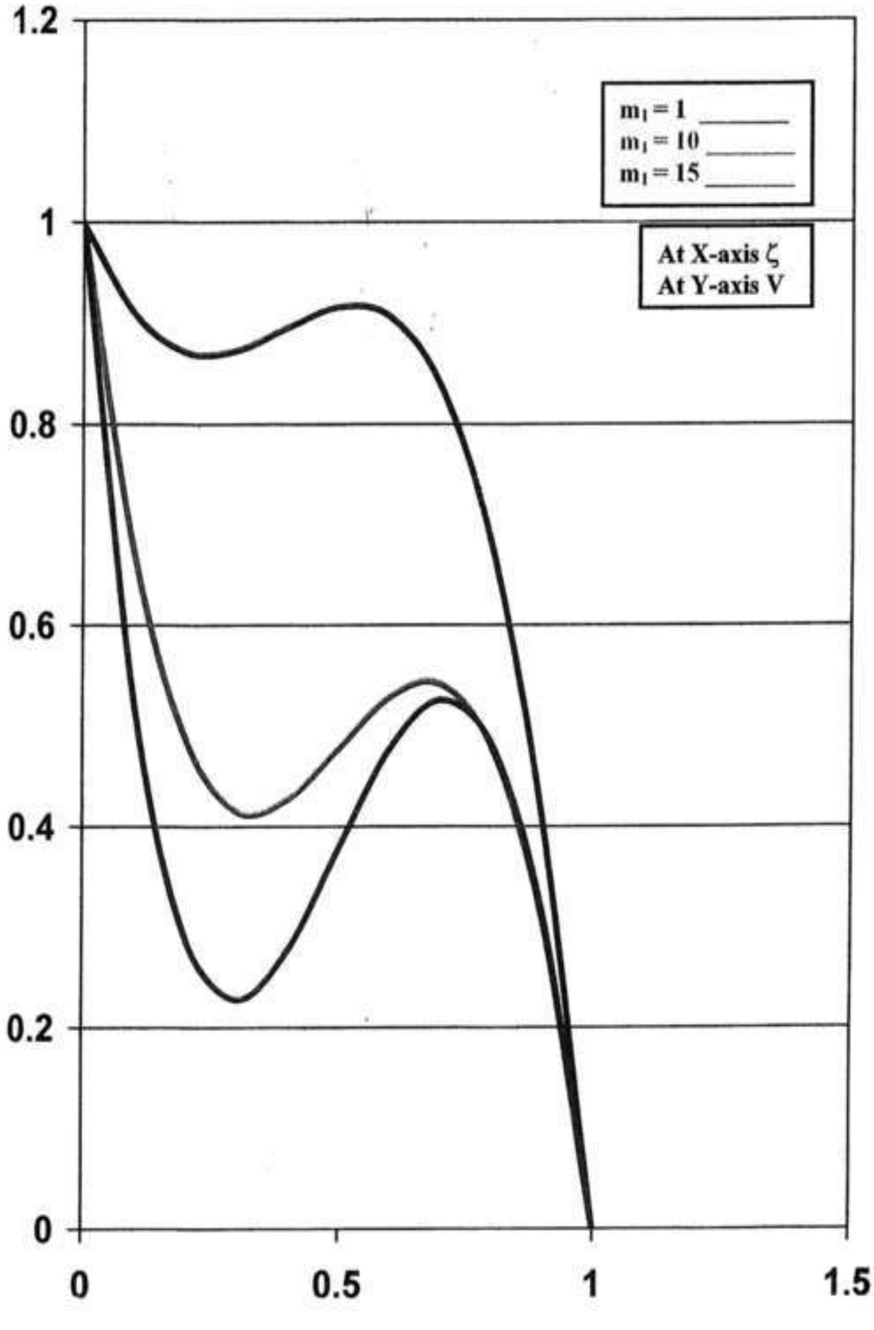
Fig(11):response of transverse velocity with different Reynolds number.



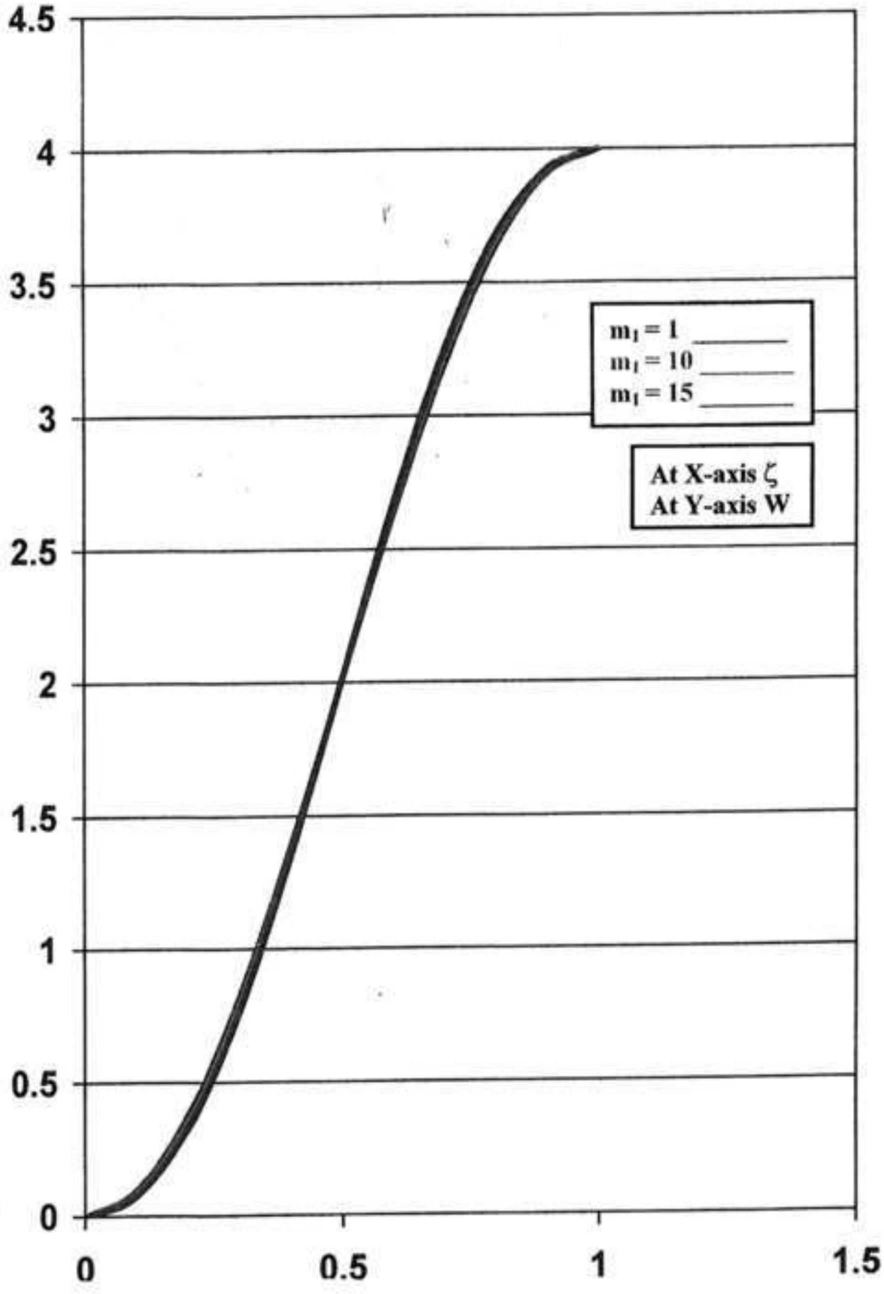
Fig(12):response of axial velocity with different Reynolds number.



Fig(13):response of radial velocity with different m_1.



Fig(14):response of transverse velocity with different m_1.



Fig(15):response of axial velocity with different m_1.

Chapter No. 3

FLOW OF A NON- NEWTONIAN SECOND-ORDER FLUID OVER AN ENCLOSED TORSINALLY OSCILLATING DISCS IN THE PRESENCE OF THE MANETIC FIELD

III.1 INTRODUCTION

The phenomenon of flow of the fluid over an enclosed torsionally oscillating disc (enclosed in a cylindrical casing) has important engineering applications. The most common practical application of it is the domestic washing machine and blower of curd etc. Soo[64)] considered first the problem of laminar flow over an enclosed rotating disc in case of Newtonian fluid. The torsional oscillations of Newtonian fluids have been discussed by Rosenblat[65)]. He has also discussed the case when the Newtonian fluid is confined between two infinite torsinally oscillating discs[66)]. Sharma & Gupta[67)] have two infinite torsionally oscillating discs. Thereafter Sharma & Singh[68)] extended the same problem for the case of porous discs subjected to uniform suction and injection. Hayat[69)]has considered non-Newtonian flows over an oscillating plate with variable suction. Chawla[70)] has considered flow past of a torsinonally oscillating plane Riley & Wybrow[71)] have considered the flow induced by the torsinally oscillations of an elliptic cylinder. Bluckburn[72)] has considered a study of two-dimensional flow past of an oscillating cylinder. Sadhna Kahre[61)] studied the steady flow between a rotating and porous stationary disc in the presence of transverse magnetic field.

Due to complexity of the differential equations and tedious calculations of the solutions of the solutions, no one has tried to solve the most practical problems of enclosed torsionally oscillating discs so far. The authors have considered the present problem of flow of a non-Newtonian second-order fluid over an enclosed torsionally oscillating disc in the presence of the magnetic field and calculated successfully the steady and unsteady part both of the flow functions. The flow functions are expanded in the powers of the amplitude $\in$ (assumed to be small) of the oscillations of the disc. The non-Newtonian effects are exhibited through two dimensionless parameters $\tau_1(=n\mu_2/n\mu_1)$ and $\tau_2(=n\mu_3/n\mu_1)$, where μ_1, μ_2, μ_3 are coefficient of Newtonian viscosity, elastic-viscosity and cross viscosity respectively, n being the uniform frequency of the oscillation. The Variation of radial, transverse and axial velocities with elastic-viscous parameter τ_1, cross-viscous parameter τ_2 Reynolds number R, magnetic field m at different phase difference τ is shown graphically.

III.2 FORMULATION OF THE PROBLEM

In the three dimensional cylindrical set of co-ordinates (r, θ,z) the system consists of a finite oscillating disc of radius r_s (coinciding with the plane z = 0) performing rotator oscillations of the type rΩCos τ of small amplitude $\in$, about the perpendicular axis r = 0 with a constant angular velocity Ω in an imcompressible second-order fluid forming the part of a cylindrical casing or housing. The top of the casing (coinciding with the plane $z = z_0 < r_z$) may be considered as a stationary disc (stator) placed parallel to and at a distance equal to gap length z_0 from the oscillating disc. The symmetrical radial steady outflow has a small mass rate 'm' of radial outflow ('-m' for net radial inflow). The inlet condition is taken as a simple radial source flow along z-axis starting from radius r_0. A constant magnetic field B_0 is applied normal to the plane of the oscillating disc. The induced magnetic field is neglected.

Assuming (u, v, w) as the velocity components along the cylindrical system of axes (r, θ, z) the relevant boundary conditions of the problem are:

$$z = 0, \qquad u = 0, \qquad v = r\,\Omega e^{i\tau}\text{(Real Part)}, \qquad w = 0,$$

$z = z_0 \qquad u = 0, \qquad v = 0, \qquad w = 0,$

where the gap z_0 is assumed small in comparison with the disc radius r_s. The velocity components for the axisymmetric flow compatible with the continuity criterion can be taken as [65,65,65].

$$U = -\xi H'(\zeta,\tau) + (R_m/R_z)\, M'(\zeta,\tau)/\xi$$

$$V = -\xi G(\zeta,\tau) + (R_L/R_z)\, L'(\zeta,\tau)/\xi$$

$$W = 2H(\zeta,\tau).$$

Where $U = u/\Omega z_0$, $V = v/\ \Omega z_0$, $W = w/\ \Omega z_0$, $\xi = r/z_0$, $+\zeta$, τ are dimensionless quantities and $H(\zeta, \tau)$, $G(\zeta,\tau)$, $L(\zeta,\tau)$, $M'(\zeta,\tau)$ are dimensionless function of the dimensionless variable $\zeta = z/z_0$ and $\tau = nt$. R_m $(={}_m/2\pi p z_0 v_1)$, R_L $(=L/2\pi p z_0 v_1)$ are dimensionless number to be called the Reynolds number of net outflow and circulatory flow respectively. R_z $(=\Omega z_0^2 v_1)$ be the flow Reynolds number. The small mass rate 'm' of the radial outflow is represented by

$$\mathbf{m} = 2\pi p \int_0^{z_0} r\, u\, dz \qquad (3.3)$$

Using expression (3.2) , the boundary conditions (3.1) transform for G, L & H into the following form:

$$G(0, \tau) = \text{Real}(e^{i\tau}),\ G(1, \tau) = 0,$$

$$L(0, \tau) = 0, \qquad L(1, \tau) = 0,$$

$$H(0, \tau) = k, \qquad H(1, \tau) = 0,$$

$H'(0,\tau) = 0, \qquad H'(1,\tau) = 0,$

(3.4)

The conditions on M on the boundaries are obtainable form the expression (3.3) for m as follows:

$M(1,\tau) - M(0,\tau) = 1$

(3.5)

which on choosing the discs as streamlines reduces to

$M(1,\tau) = 1, \qquad M(0,\tau) = 0$

(3.6)

Using eqs. (1.4) and expression (3.2) in equation (1.8) and neglecting the squares

& higher powers of R_m/R_z (assumed small), we have the following equations in dimensionless form:

$-(1/pz_0)(\partial p/\partial \xi) = -n\Omega z_0\{\xi\partial H' - (R_m/R_z)(\partial M'/\xi)\} + \Omega^2 z_0\xi(H'^2 - 2HH'' - G^2) + \Omega^2 z_0 \quad (R_m/R_z)(2HM''\xi) - \Omega^2 z_0(R_L/R_z)(2LG/\xi) + (v_1\Omega/z_0)\{H'''\xi - (R_m/R_z)(M'''/\xi)\} - (2v_2/z_0)\,[n\Omega/2]\,\{(R_m/R_z)\,(\partial M'''/\xi) - \xi\partial H'''\} + \Omega^2 \xi(H''^2 - HH^{iv}) + (R_m/R_z)\,(\Omega^2/\xi)\,(H'''M' + H''M'' + H'M''' + HM^{iv}) - (R_L/R_z)(2\Omega^2/\xi)(L'G' + LG'')] - (4v_3\Omega^2 - z_0)\{(R_m/R_z)\,(1/2\xi)\,(H'''M' + H'M''' + H''M'') - (R_L/R_z)(1/2\xi)(2L'G' + LG'') + (\xi/4)\,(H''^2 - G'^2 - 2H'H''')\} + (\sigma B_0^2\Omega z_0/p)\{-\xi H' + (R_m/R_z)(M'/\xi)\}.$

(3.7)

$0 = n\Omega z_0\{\xi\partial G + (R_L/R_z)(\partial L/\xi)\} - (2\Omega^2 z_0\xi)(HG' - H'G) - \quad \Omega^2 z_0(R_m/R_z)\,(2M'G/\xi) - \Omega^2 z_0(R_L/R_z)(2HL'/\xi) + (v_1\Omega/z_0)\{\xi G'' + (R_L/R_z)(L''/\xi\,)\} + (2v_2/z_0)[(n\Omega/2)\quad\{\xi\partial G'' + (R_L/R_z)(\partial L''/\xi)\quad\} + \quad (R_L/R_z)\quad(\Omega^2/\xi)$

$(H''L'+H'''L+HL'''+H'L'')+(\Omega^2/\xi)(HG'''$

$H''G')+(R_m/R_z)(2\Omega^2/\xi)(M'G''+M''G')]+(2v_3\Omega^2/z_0)\{\xi(H'G''-H''G')$

$+(R_L+R_z)(1/\xi)(H''L'+H'''L+H'L'')+(R_m+R_z)(1/\xi)(2M''G'+M'G'')$

$-(\sigma B_0{}^2\Omega z_0/p) \quad \{\xi G+(R_L/R_z)(L/\xi)\}.$

(3.8)

$-(1/pz_0)(\partial p/\partial \zeta)=2n\Omega z_0\partial H+4\Omega^2 z_0 HH'-2v_1\Omega H''/z_0-$

$(2v_2/z_0)\{n\Omega\partial H''+2\Omega^2\xi^2 (H''H'''+G'G'')+ \Omega^2(22H'HH''+2HH''')-$

$(R_m/R_z)2\Omega^2(H''M'''+H'''M'')+ (R_L/R_z) 2\Omega^2 (L'G'' + L''G')\}-(2v_3$

$\Omega^2/z_0)\{\xi^2(H''H''' +G'G'')+14H'H''-(R_m/R_z)$

$(H''M'''+H'''M'')+(R_L/R_z)(L'G''+L''G')\}$

(3.9)

where B_0 and σ are intensity of the magnetic field and conductivity of the fluid considered. R $(=z_0^2/v_1)$ is the Reynolds number, τ_1 $(=nv_2/v_1)$, τ_2 $(=nv_3/v_1)$ and $\in(=\Omega/n)$ are the dimensionless parameter, $m^2 = \sigma B_0^2 z_0^2/\mu_1$ is the dimensionless magnetic field and R_m/R_L= **m/L**$\approx$1.

For $R_m = R_L = B_0 = 0$, the differential equations (3.7)-(3.9) are identical to those obtained by Sharma and Gupta[67)] (for $S_1 = 1$, $S_2 = 0$) and differentiating (3.7) w.r.t ζ and (3.9) w.r.t ξ and then eliminating $\partial^2 p/\partial\zeta.\partial\xi$ from the equations thus obtained. We get

$-n\Omega z_0\{\xi\partial H''-(R_m/R_z)\partial M''/\xi\}-2\Omega^2 z_0\xi(HH'''-GG')+(R_m/R_z)(2\Omega^2 z_0/\xi)$

$(H'M''+HM''')-(R_L/R_z)(2\Omega^2 z_0/\xi)(LG'+L'G)$

$(v_1\Omega/z_0)\{(R_m/R_z)(M^{iv}/\xi)- \xi H^{iv}\}-(2v_2/z_0) [(n\Omega/2)\{(R_m/R_z)(\partial M^{iv}/\xi)-$

$\xi\partial H^{iv}\}-\Omega^2\xi(2H''H'''+H'H^{iv}+HH^{v}+4G'G'')$

$+(R_m/R_z)(\Omega^2/\xi)(2H'''M''+H^{iv}M'+2H''M'''+2H'M^{iv}+HM^{v})-$

$(R_L/R_z)(2\Omega^{2/}\xi)(2L'G''+L''G'+LG''')]-(2v_3\Omega^2/z_0)\{(R_m+R_z)(1/\xi)$

$(H^{iv}M'+2H'''M''+2H''M'''+H'M^{iv})(R_L+R_z)(1/\xi)$

$(3L'G''+2L''G'+LG''')$– $\xi(H'H^{iv}+3G'G''+2H''H''')\} + (\sigma B_0{}^2\Omega z_0/p)$ -
$\{\xi H+(R_m/R_z)(M''/\xi)\}=0$

(3.10)

On equating the coefficients of ξ and $1/\xi$ from the equation (3.8) & (3.10), we get the following equations:

$G''=R\partial G+2\in R(HG'-H'G)-\tau_1\partial G''-2\in \tau_1(HG'''-H''G')-2\in$
$\tau_2(H'G''H''G')+m^2G$

(3.11)

$L''= R\partial L+2\in R(M'G+HL)-\tau_1\partial L''-2\in$
$\tau_1(H''L'+H'''L+HL'''+H'L''+2M'G''+2M''G')-2\in$
$\tau_2(H''L'+H'''L+H'L''+2M''G'+M'G')+m^2L$

(3.12)

$H^{iv}=R\partial H+2\in R(HH'''+GG')-\tau_1\partial H^{iv}-2\in$
$\tau_1(H'H^{iv}+HH^{v}+2H''H'''+4G'G'')-2\in$
$\tau_2(H'H^{iv}+2H''H''+3G'G'')+m^2H''$

(3.13)

$M^{iv}=R\partial M+2\in R(H'M''+HM'''-LG'-L'G)-\tau_1\partial M^{iv}-2\in$
$\tau_1(2H'''M''+H^{iv}M'+2H''M''' +2H'M^{iv}+HM^{v}-4L'G''-2L''G'-2LG''')-2\in \tau_2(H^{iv}M'+2H'''M''+2H''M'''+H'M^{iv}-3L'G''-2L''G'-LG''')+m^2M''$

(3.14)

III.3 SOLUTION OF THE PROBLEM

Substituting the expressions

$G(\zeta,\tau) = \sum \in^N G_N(\zeta,\tau)$

$L(\zeta,\tau) = \sum \in^N L_N(\zeta,\tau)$

$H(\zeta,\tau) = \sum \in^N H_N(\zeta,\tau)$

$M(\zeta,\tau) = \sum \in^N M_N(\zeta,\tau)$

(3.15)

into (3.11) to (3.14) neglecting the terms with coefficient of $\in^2$ (assumed negligible small) and equating the terms independent of $\in$ and coefficient of $\in$, we get the following equations:

$G_0'' = R\, \partial G_0/\partial\tau - \tau_1\, \partial G_0''/\partial\tau + m^2 G_0$

(3.16)

$G_1'' = R\, \partial G_1/\partial\tau - 2R(H_0' G_0' - H_0 G_0') - \tau_1\, \partial G_1''/\partial\tau - 2\tau_1(H_0' G_0''' - H_0'' G_0') - 2\tau_2(H_0' G_0'' - H_0'' G_0') + m^2 G_1$

(3.17)

$L_0'' = R\, \partial L_0/\partial\tau - \tau_1\, \partial L_0''/\partial\tau + m^2 L_0$

(3.18)

$L_1'' = R\, \partial L_1/\partial\tau - 2R(M_0' G_0' - H_0 L_0') - \tau_1\, \partial L_1''/\partial\tau - 2\tau_1(H_0''' L_0 + H_0'' L_0' + H_0' L_0'' + H_0 L_0'''$

$+ 2M_0'' G_0' + 2M_0' G_0'') - 2\tau_2(H_0''' L_0 + H_0'' L_0' + H_0' L_0'' + 2M_0'' G_0' + M_0' G_0'') + m^2 L_1$

(3.19)

$H_0^{iv} = R\partial H_0''/\partial\tau - \tau_1 \partial H_0^{iv}/\partial\tau + m^2 H_0''$

(3.20)

$H_1^{iv} = R\, \partial H_1''/\partial\tau + 2R(H_0 H_0''' + G_0 G_0') - \tau_1\, \partial H_1^{iv}/\partial\tau - 2\tau_1(H_0' H_0^{iv} + H_0 H_0^{v} + 2H_0'' H_0''' +$

$4G_0'G_0'')- 2\tau_2(3G_0'G_0''+H_0'H_0^{iv}+2H_0''H_0''')+ m^2H_1''$

(3.21)

$M_0^{iv}=R\partial M_0''/\partial\tau-\tau_1\partial M_0^{iv}/\partial\tau+m^2M_0''$

(3.22)

$M_1^{iv}=R\partial M_1''/\partial\tau-2R(H_0'M_0''+H_0M_0'''-L_0'G_0-L_0G_0')- \tau_1\partial M_1^{iv}/\partial\tau-2\tau$

$_1(2H_0'''M_0''$

$+H_0^{iv}M_0'+2H_0''M_0'''-4L_0'G_0''-2L_0''G_0'-2L_0G_0''')+H_0M_0^{v}+2H_0'M_0^{iv})-2\tau_2$

$(2H_0'''M_0'' +H_0^{iv}M_0'+2H_0''M_0 '''-3L_0'G_0''-2L_0''G_0'-L_0G_0'''+H_0'M_0^{iv})+ m^2M_1$ (3.23)

Taking $G_n(\zeta,\tau) = G_{ns}(\zeta)+e^{i\tau}G_{nt}(\zeta)$

$L_n(\zeta,\tau) =L_{ns}(\zeta)+e^{i\tau}L_{nt}(\zeta)$

$H_n(\zeta,\tau) = H_{ns}(\zeta)+e^{2i\tau}H_{nt}(\zeta)$

$M_n(\zeta,\tau) = M_{ns}(\zeta)+e^{2i\tau}M_{nt}(\zeta)$

(3.24)

Complex notation has been adopted here with the convention that only real pats of the complex quantities have the physical meaning.

Using (6.24) and (6.33), the boundary conditions (6.5) & (6.7) for n = 0,1 transform to

$G_{0s}(0) = 0,\quad G_{0t}(0) = 1,\qquad G_{1s}(0) = 0,\quad G_{1t}(0) = 0,$

$G_{0s}(1) = 0,\quad G_{0t}(0) = 0,\qquad G_{1s}(1) = 0,\quad G_{1t}(1) = 0,$

$H_{0s}(0) = k,\quad H_{0t}(0) = 0,\qquad H_{1s}(0) = 0,\quad H_{1t}(0) = 0,$

$H_{0s}(1) = k,\quad H_{0t}(1) = 0,\qquad H_{1s}(1) = 0,\quad H_{1t}(1) = 0,$

$H'_{0s}(0) = 0,\ H'_{0t}(0) = 0,\quad H'_{1s}(0) = 0,\ H'_{1t}(0) = 0,$

$H'_{0s}(1) = 0,\ H'_{0t}(1) = 0,\quad H'_{1s}(1) = 0,\ H'_{1t}(1) = 0,$

$L_{0s}(0) = 0,\ L_{0t}(0) = 0,\quad L_{1s}(0) = 0,\ L_{1t}(0) = 0,$

$L_{0s}(1) = 0,\ L_{0t}(1) = 0,\quad L_{1s}(1) = 0,\ L_{1t}(1) = 0,$

$M'_{0s}(0) = 0,\ M'_{0t}(0) = 0,\quad M'_{1s}(0) = 0,\ M'_{1t}(0) = 0,$

$M_{0s}(0) = 0,\ M_{0t}(0) = 0,\quad M_{1s}(0) = 0,\ M_{1t}(0) = 0,$

$M_{0s}(0) = 0,\ M_{0t}(0) = 0,\quad M_{1s}(0) = 0,\ M_{1t}(0) = 0,$

$M_{0s}(1) = 0,\ M_{0t}(1) = 0,\quad M_{1s}(1) = 0,\ M_{1t}(1) = 0,$

(3.25)

III.4 RESULTS AND DISCUSION

The variation of the radial velocity with ζ at τ_2= 2, ξ = 5, R = 5, R_m= 0.05, R_L = 0.049, R_z = 2, m = 2 for different values of elastic-viscous parameter τ_1= 0, -0.3 and phase difference τ= $\pi/3$, $2\pi/3$ is shown in fig (1). For τ= $\pi/3$, the radial velocity increases with an increase in ζnear the lower disc, attains its maximum value at ζ=0.2then start decreasing, attain its minimum value at ζ=0.8 and increases thereafter near the upper disc. It is clear that the radial velocity increases with an increase in τ_1 near the lower disc then start decreasing with an increase in τ_1 after the point of intersection near the upper disc. For τ =$2\pi/3$, the radial velocity increases with an increase in ζ and start decreasing thereafter at τ_1=0 whenever at τ_1 = -0.3 it decreases first, attains its minimum value at ζ = 0.1then start increasing, attains its maximum value at $\zeta =$ 0.7and decreases there after upto the surface of the upper disc. It is also seen from this figure that the radial velocity increases with an increase in τ_1 upto the middle of the gap-length and decreases thereafter with an increase in τ_1 upto the surface of the upper disc.

The variation of the transverse velocity with ζ at τ_2= 2, ξ = 5, R = 5, R_m= 0.05, R_L = 0.049, R_z = 2, m = 2 for different values of elastic-viscous parameter τ_1= 0, -0.3 and phase difference τ= π/3, 2π/3 is shown in fig (2). For τ= π/3, the transverse velocity decreases with an increase in ζ. It is observed from this figure that velocity decreases with an increase in τ_1 throughout the gap-length. It is also seen that transverse velocity is maximum at lower disc minimum at the upper disc. For τ =2π/3, the transverse velocity increases with an increase in ζ whenever it start decreasing near the upper disc. It is also observed that the transverse velocity decreases with an increase in τ_1 throughout the gap-length and is minimum at lowe disc.

The variation of the axial velocity with ζ at τ_2= 2, ξ = 5, R = 5, R_m= 0.05, R_L = 0.049, R_z = 2, m = 2 for different values of elastic-viscous parameter τ_1= 0, -0.3 and phase difference τ= π/3, 2π/3 is shown in fig (3). For τ= π/3, the axial velocity decreases with an increase in ζ, attains its minimum value in the middle of the gap length approximately and increases thereafter upto the surface of the upper disc. It is clear from this figure that the axial velocity decreases with an increase in τ_1 upto the middle of the gap-length from the lower disc approximately whenever its behavior is reversed thereafter. For τ= 2π/3, the axial velocity increases with an increase in ζ near the lower disc then start decreasing upto ζ= 0.65 and start increasing thereafter upto the upper disc τ_1= 0 whenever at τ_1= -0.3, the axial velocity increases with an increases in $\zeta\ upto\ \zeta$ = 0.4 and decreases thereafter. It is also evident from this figure that the axial velocity decreases with an increase in τ throughout the gap-length.

The variation of the radial velocity with ζ at τ_1= -2, ξ = 5, R = 5, R_m= 0.05, R_L = 0.049, R_z = 2, m = 2 for different values of cross-viscous parameter τ_2= 0, 15 and phase difference τ= π/3, 2π/3 is shown in fig (4). For τ= π/3, the graph of the radial velocity is parabolic with vertex upward with maximum value at ζ=0.5. It is seen from this figure the radial velocity increases with an increase in τ_2 throughout the gap-length. For τ =2π/3, the radial velocity decreases with an increase in ζ, attain its minimum value at ζ = 0.6 and increases thereafter in case of τ_2= 0 whenever the graph of velocity is parabolic with vertex upward with maximum value at $\zeta =$ 0.5 in case of τ_2= 15. It is also observed from this figure that the radial velocity increases with an increase in τ_2 throughout the gap-length.

The variation of the transverse velocity with ζ at τ_1= -2, ξ = 5,R= 5, R_m= 0.05, R_L = 0.049, R_z = 2, m = 2 for different values of cross-viscous parameter τ_1= 0, 15 and phase difference τ= π/3, 2π/3 is shown in fig (5). For τ= π/3, the transverse velocity decreases with an increase in ζ, throughout the gap-length. It is evident from this figure that the transverse velocity is being overlapped throughout the gap-length for all the values of τ_2. For τ=2π/3, the transverse velocity increases with an increase in ζ throughout the gap-length and is also being overlapped for both values of τ_2.

The variation of the axial velocity with ζ at τ_1= -2, ξ = 5,R= 5, R_m= 0.05, R_L = 0.049, R_z = 2, m = 2 for different values of cross-viscous parameter τ_2= 0, 15 and phase difference τ= π/3, 2π/3 is shown in fig (6). For τ= π/3, the axial velocity increases with an increase in ζ near the lower disc attain its maximum value in the middle of the second half and again increases upto the upper disc at τ_2= 0 whenever at τ_2= 15, the axial velocity decreases with an increase in ζ, attain the its minimum value at ζ=0.7approximately and start increasing, attain its value at ζ=0.7 approximately and again decreases upto the upper disc at τ_2= 0 whenever at τ_2= 15, the graph of the axial velocity is parabolic with vertex upward with maximum value ζ=0.5. It is also seen from this figure that the axial velocity increases with an increase in τ_2 throughout the gap-length.

The variation of the radial velocity with ζ at τ_1= -2, ξ = 5, τ_2= 10, R_m= 0.05, R_L = 0.049, R_z = 2, R = 5 for different values of magnetic field parameter m = 10, 15 and phase difference τ= π/3, 2π/3 is shown in fig (7). For τ= π/3, the radial velocity increases with an increase in ζ near the lower disc attain its maximum value at ζ = 0.2 then start decreasing, attains its minimum value at ζ = 0.8 and increase upto the surface upper disc. It is evident from this figure that the radial velocity increases with an increase in m near the lower disc while its behavior is reversed after the point of intersection of both branches. For τ =2π/3, the graph of the radial velocity is just reversed to that graph at τ =π/3. It is also seen from this figure that the radial velocity decreases with an increase in m near the lower and upper discs while it increases with an increase in m in the middle two two quarters.

The variation of the transverse velocity with ζ at τ_1= -2, ξ = 5, τ_2= 10, R_m= 0.05, R_L = 0.049, R_z = 2, R = 5 for different values of magnetic field parameter m = 10, 15 and phase difference τ= π/3, 2π/3 is shown in fig (8). For τ= π/3, the transverse velocity decreases with an increase in magnetic field parameter m throughout the gap-length. It is also clear that the transverse velocity is maximum at lower disc and minimum at the upper disc. For τ = 2π/3, the transverse velocity increases with an increase in ζ first and then start decreasing upto the surface of the upper disc. It is also clear from this figure that the velocity increases with an increase in m up to ζ = 0.7 whenever it decreases with an increase in m after the point of intersection of both braches near the upper disc.

The variation of the transverse velocity with ζ at τ_1= -2, ξ = 5, τ_2= 10, R_m= 0.05, R_L = 0.049, R_z = 2, R = 5 for different values of magnetic field parameter m = 10, 15 and phase difference τ= $\pi/3$, $2\pi/3$ is shown in fig (9). For τ= $\pi/3$, the axial velocity decreases with an increase in ζ upto middle of the gap-length approximately and increase in ζ upto the surface of the a upper disc. It is evident from this figure that the axial velocity decreases with an increase in m throughout the gap-length. For τ = $2\pi/3$, the graph of the radial velocity is increase ζ = 0.4 approximately and then decreases upto the surface of the upper disc. It is seen from this figure that the axial velocity increases with an increase in m upto ζ = 0.6 whenever it decreases with an increase in m after the point of intersection near the upper disc.

The variation of the radial velocity with ζ at τ_1= -2, ξ = 5, τ_2= 10, R_m= 0.05, R_L = 0.049, R_z = 2, R = 5 for different values of Reynolds number R = 2, 6 and phase difference τ= $\pi/3$, $2\pi/3$ is shown in fig (10). For τ= $\pi/3$, the graph of the radial velocity is parabolic with vertex downward with minimum value at ζ = 0.5 in case R =2 whenver the graph of the radial velocity at R = 6 is just reversed to that of graph at R = 2. It is observed from this figure that the radial velocity increases with an increase in R throughout the gap-length. For τ = $2\pi/3$, the behavior of the radial velocity is same to that of at τ = $\pi/3$.

The variation of the transverse velocity with ζ at τ_1= -2, ξ = 5, τ_2= 10, R_m= 0.05, R_L = 0.049, R_z = 2, R = 5 for different values of Reynolds number R = 2, 6 and phase difference τ= $\pi/3$, $2\pi/3$ is shown in fig (11). For τ= $\pi/3$, the transverse velocity decreases with an increase in ζ . It is clear from this figure that the transverse velocity increases with an increase in R throughout the gap-length. The transverse velocity is maximum at lower disc and minimum at the upper disc. For τ = $2\pi/3$, the transverse velocity increases with an increase in ζ .It is also seen from this figure that the transverse velocity is being overlapped for all the values of R. The transverse velocity is minimum at lower disc and maximum at the upper disc.

The variation of the axial velocity with ζ at τ_1= -2, ξ = 5, τ_2= 10, R_m= 0.05, R_L = 0.049, R_z = 2, R = 5 for different values of Reynolds number R = 2, 6 and phase difference τ= π/3, 2π/3 is shown in fig (12). For τ= π/3, axial velocity increases with an increase in ζ, attain its maximum value at ζ = 0.7 and decreases upto surface of the upper disc at R = 2 whenever at R = 6, the axial velocity increases upto ζ = 0.1 then start decreasing, attains its minimum value at ζ = 0.7 and increases upto the surface of the upper disc. It is clear from this figure that the axial velocity decreases with an increases in R throughout the gap length. For τ =2π/3, the axial velocity decreases with an increase in ζ upto ζ = 0.25 then start increasing, attains its maximum value at ζ = 0.8 and decreases upto the upper disc at R = 2 whenever at R = 6, the axial velocity increases with an increase in ζ upto ζ = 0.6 and decreases upto surface of the upper disc. It is evident this figure that the axial velocity increases with an increase R throughout the gap-length.

	τ= π/3		τ= 2π/3	
ζ	τ_1= 0	τ_1= -0.3	τ_1= 0	τ_1= -0.3
0.0	0.000000	0.000000	0.000000	0.000000
0.1	0.019381	0.015124	0.000669	-0.008994
0.2	0.021618	0.020329	0.005575	-0.007345
0.3	0.016271	0.018487	0.009415	-0.001144
0.4	0.008792	0.012601	0.101728	0.005785
0.5	0.009135	0.005437	0.009842	0.011343
0.6	-0.003127	-0.001016	0.007701	0.014573
0.7	-0.006020	-0.005392	0.005245	0.015155
0.8	-0.006629	-0.006879	0.003108	0.013016
0.9	-0.004771	-0.005096	0.001474	0.008060

1.0	0.000000	0.000000	0.000000	0.000000
	Table (1) variation Radial velocity U			

	$\tau = \pi/3$		$\tau = 2\pi/3$	
ζ	$\tau_1 = 0$	$\tau_1 = -0.3$	$\tau_1 = 0$	$\tau_1 = -0.3$
0.0	2.500000	2.500000	2.500000	2.500000
0.1	2.332912	2.481212	-1.624245	-1.608388
0.2	2.097773	2.341571	-0.998594	-0.951595
0.3	1.828391	2.122754	-0.564612	-0.486259
0.4	1.546827	1.855166	-0.275867	-0.174522
0.5	1.266489	1.560127	-0.095709	0.016203
0.6	0.994486	1.251759	0.004676	0.113724
0.7	0.733409	0.938600	0.047957	0.141723
0.8	0.482676	0.624997	0.052156	0.120520
0.9	0.239562	0.312313	0.031973	0.067901
1.0	0.000000	0.000000	0.000000	0.000000
	Table (2) variation Transverse velocity V			

	$\tau = \pi/3$		$\tau = 2\pi/3$	
ζ	$\tau_1 = 0$	$\tau_1 = -0.3$	$\tau_1 = 0$	$\tau_1 = -0.3$
0.0	0.000000	0.000000	0.000000	0.000000
0.1	-0.020437	-0.014122	0.003737	0.014449
0.2	-0.055658	-0.043344	0.005411	0.039925
0.3	-0.083135	-0.071888	0.001375	0.060294
0.4	-0.094955	-0.089908	-0.005540	0.069388
0.5	-0.091106	-0.093274	-0.011525	0.066937
0.6	-0.075289	-0.082472	-0.14114	0.055642
0.7	-0.52612	-0.061297	-0.012779	0.039334

0.8	-0.028586	-0.035669	-0.008623	0.022031
0.9	-0.008952	-0.012698	-0.003675	0.007625
1.0	0.000000	0.000000	0.000000	0.000000
	Table (3) variation axial velocity w			

	$\tau = \pi/3$		$\tau = 2\pi/3$	
ζ	$\tau_2 = 0$	$\tau_2 = 15$	$\tau_2 = 0$	$\tau_2 = 15$
0.0	0.000000	0.000000	0.000000	0.000000
0.1	0.009095	0.021820	-0.002802	0.008488
0.2	0.016642	0.040748	-0.005683	0.015256
0.3	0.022458	0.055866	-0.008276	0.020290
0.4	0.026355	0.066363	-0.010273	0.023546
0.5	0.028148	0.071534	-0.011428	0.024928
0.6	0.027661	0.070778	-0.011552	0.024319
0.7	0.024734	0.063598	-0.010514	0.021606
0.8	0.019226	0.049595	-0.008241	0.016689
0.9	0.011015	0.028466	-0.004720	0.009499
1.0	0.000000	0.000000	0.000000	0.000000
	Table (4) variation Radial velocity U			

	$\tau = \pi/3$		$\tau = 2\pi/3$	
ζ	$\tau_2 = 0$	$\tau_2 = 15$	$\tau_2 = 0$	$\tau_2 = 15$
0.0	2.500000	2.500000	-2.500000	-2.500000
0.1	2.332013	2.337302	-2.200005	-2.190833
0.2	2.138674	2.145357	-1.914615	-1.902344
0.3	1.922525	1.928038	-1.642790	-1.631929
0.4	1.686208	1.689064	-1.383199	-1.376860
0.5	1.432440	1.432040	-1.134551	-1.134287
0.6	1.163999	1.160506	-0895314	-0.901238

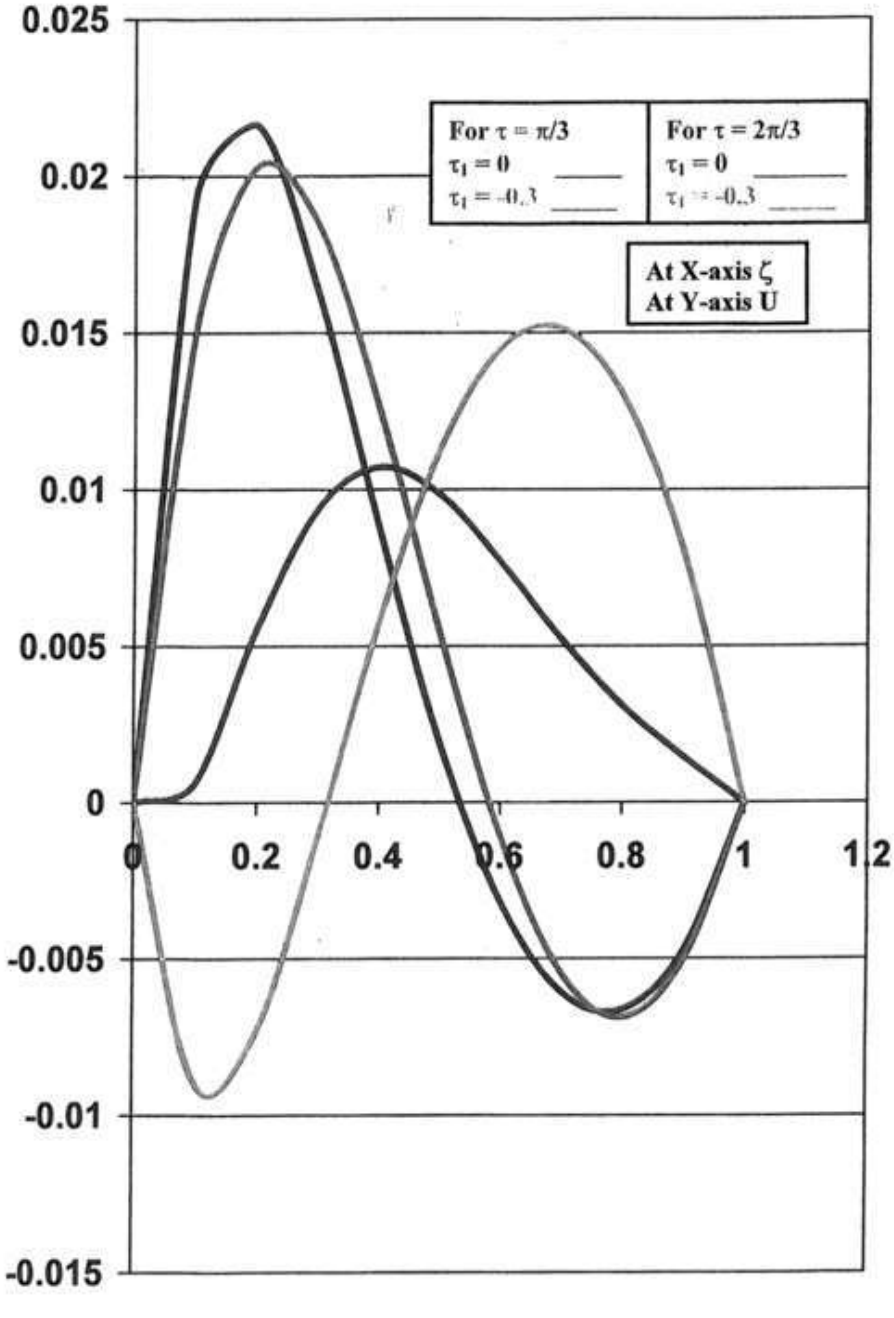
0.7	0.883698	0.877977	-0.663901	-0.674610
0.8	0.594373	0.587980	-0.438664	-0.451159
0.9	0.298864	0.294095	-0.217923	-0.227482
1.0	0.000000	0.000000	0.000000	0.000000
	Table (5) variation of Transverse velocity V			

	$\tau = \pi/3$		$\tau = 2\pi/3$	
ζ	$\tau_2 = 0$	$\tau_2 = 15$	$\tau_2 = 0$	$\tau_2 = 15$
0.0	0.000000	0.000000	0.000000	0.000000
0.1	0.014847	-0.004646	-0.000766	0.056482
0.2	0.020154	-0.033140	0.005697	0.1000013
0.3	0.017647	-0.078639	0.017446	0.130735
0.4	0.009480	-0.132504	0.032033	0.148739
0.5	-0.001824	-0.184615	0.046588	0.154142
0.6	-0.013431	-0.223677	0.0570906	0.147122
0.7	-0.022281	-0.237540	0.062538	0.127922
0.8	-0.025165	-0.213500	0.056878	0.096824
0.9	-0.018816	-0.138613	0.037250	0.054109
1.0	0.000000	0.000000	0.000000	0.000000
	Table (6) Variation of Axial velocity W			

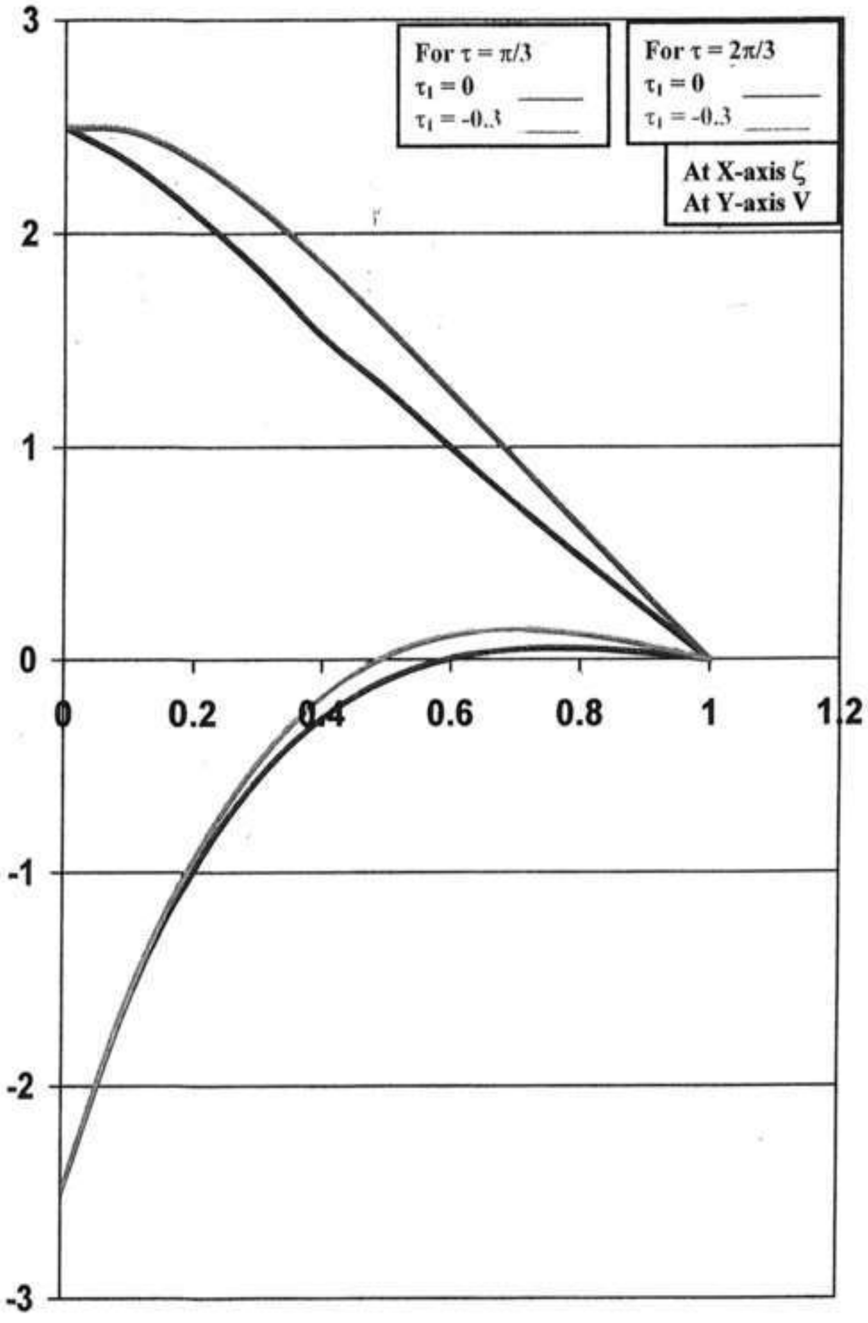
	$\tau = \pi/3$		$\tau = 2\pi/3$	
ζ	m = 10	m = 15	m = 10	m = 15
0.0	0.000000	0.000000	0.000000	0.000000
0.1	0.015938	0.033323	-0.021138	-0.033708
0.2	0.022353	0.039273	-0.021606	-0.026780
0.3	0.020279	0.028033	-0.011398	-0.007892
0.4	0.013005	0.011043	0.002092	0.009297
0.5	0.004058	-0.004372	0.014280	0.020631

0.6	-0.003729	-0.014753	0.022714	0.026123
0.7	-0.008458	-0.018977	0.026241	0.026848
0.8	-0.009160	-0.017047	0.024272	0.023513
0.9	-0.005853	-0.009750	0.016099	0.015663
1.0	0.000000	0.000000	0.000000	0.000000
	Table (7) Variation of Radial velocity R			

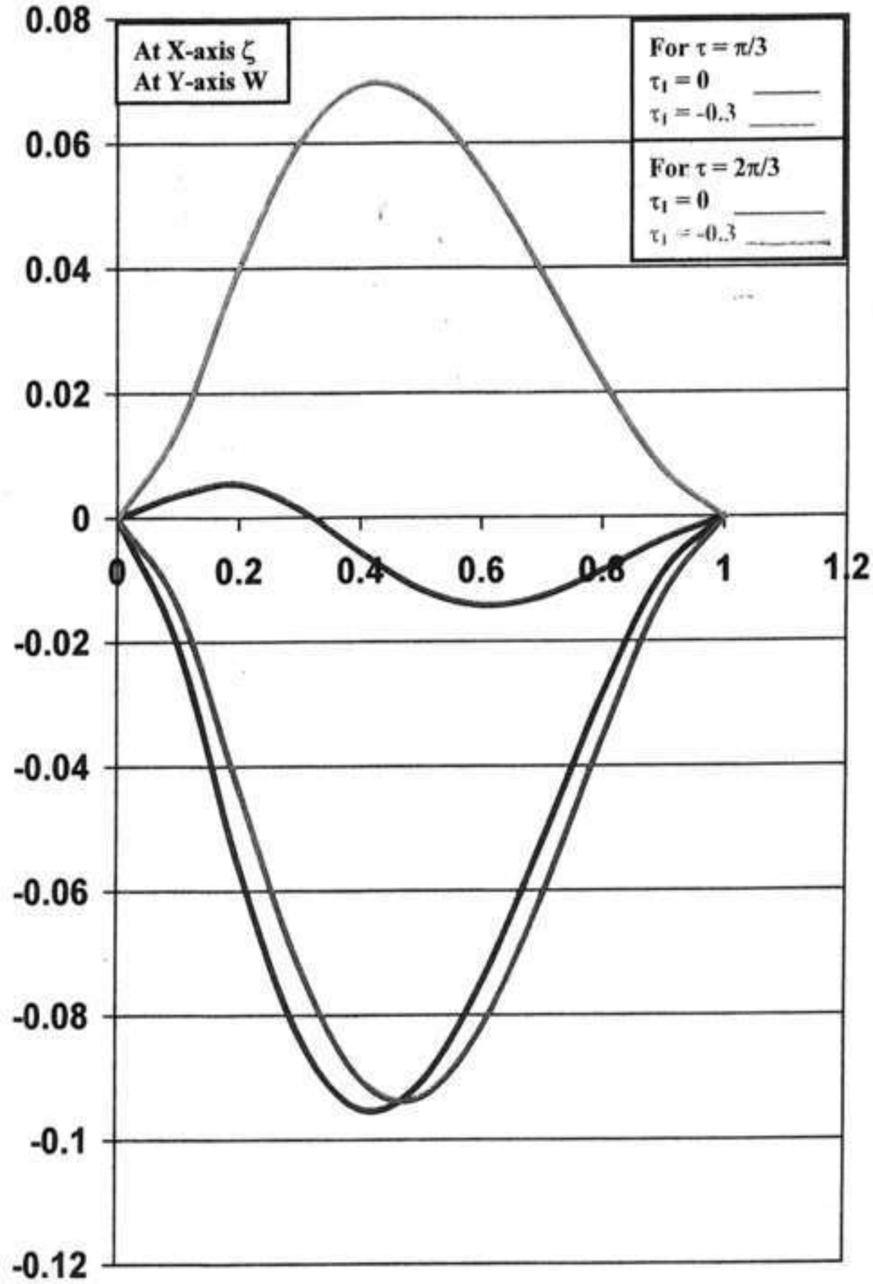
	$\tau = \pi/3$		$\tau = 2\pi/3$	
ζ	m = 10	m = 15	m = 10	m = 15
0.0	2.500000	2.500000	2.500000	2.500000
0.1	2.473640	2.372010	-1.382090	-0.951500
0.2	2.279813	1.980891	-0.621527	0.098088
0.3	1.998898	1.525220	-0.131262	0.314663
0.4	1.682168	1.103490	0.127873	0.463619
0.5	1.361051	0.756462	0.300526	0.466883
0.6	1.052834	0.492271	0.339199	0.398727
0.7	0.764716	0.302109	0.306424	0.302200
0.8	0.496833	0.169548	0.226888	0.199235
0.9	0.244528	0.075472	0.119573	0.098303
1.0	0.000000	0.000000	0.000000	0.000000
	Table (8) Variation of Transverse velocity V			



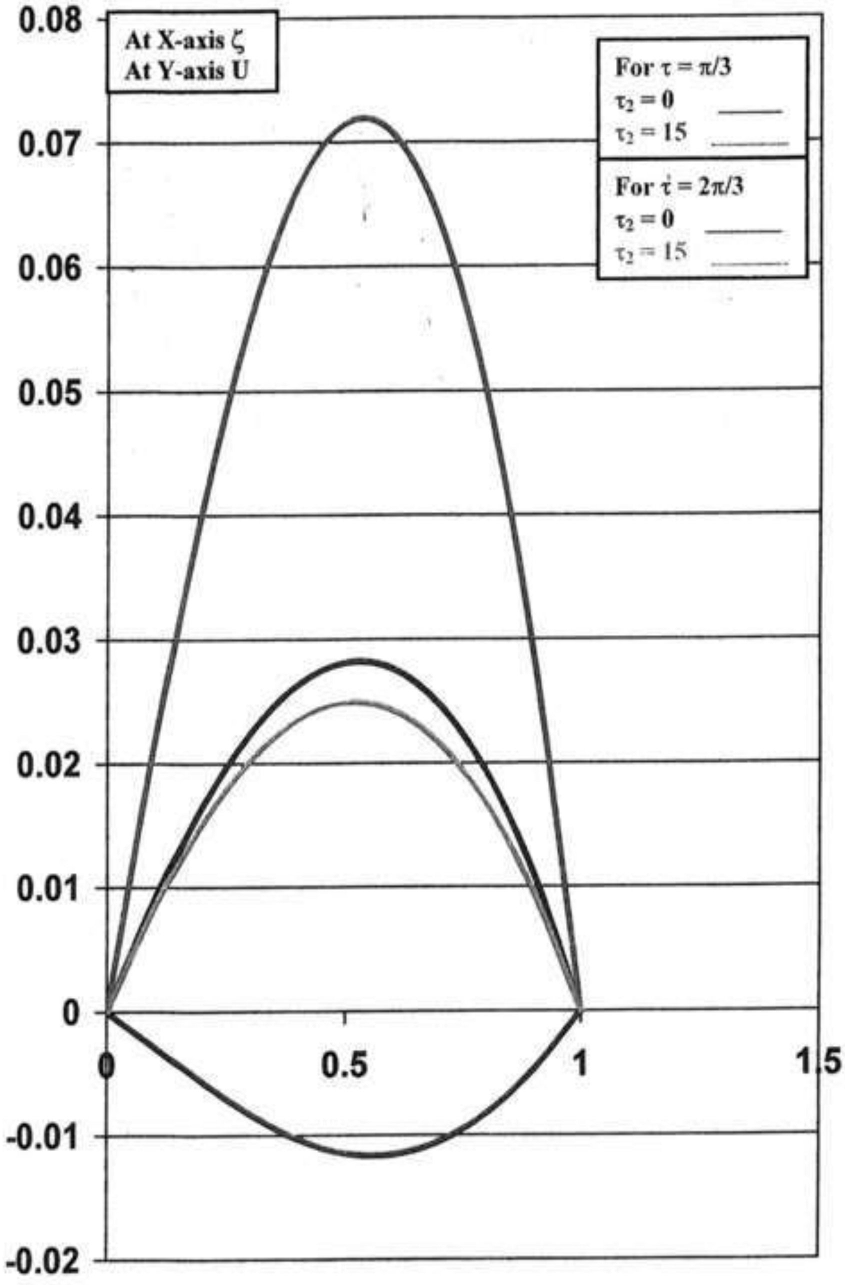
Fig(1) Variation of radial velocity U.



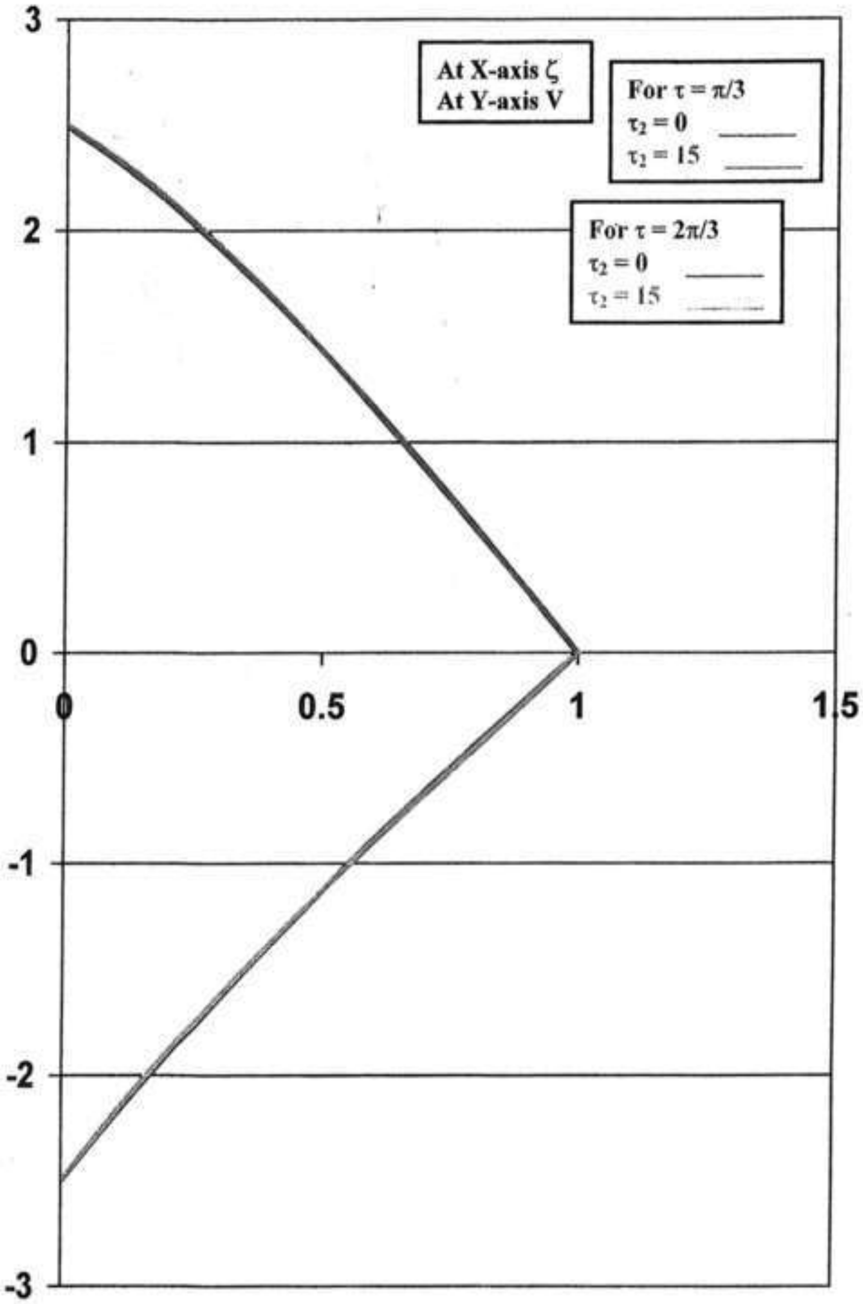
Fig(2) Variation in transverse velocity V.



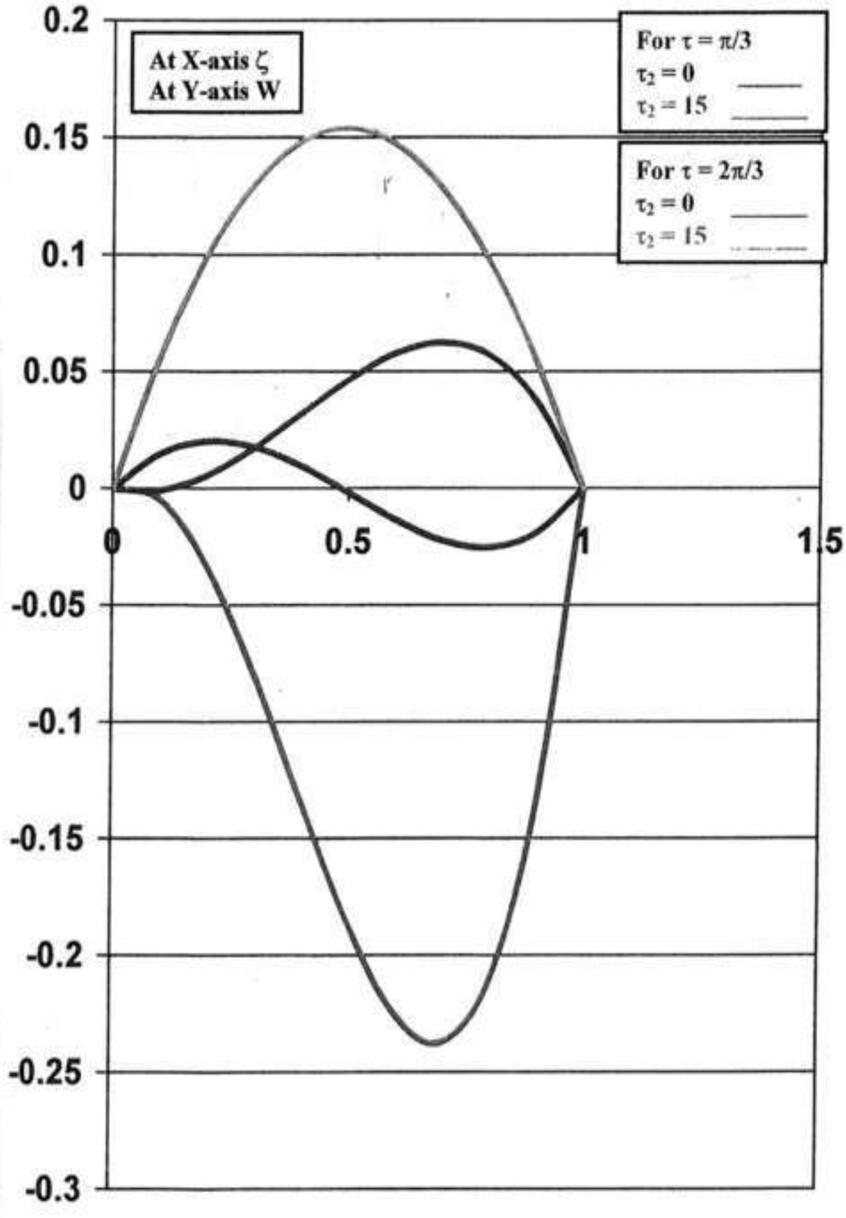
Fig(3) Variation in axial velocity W.



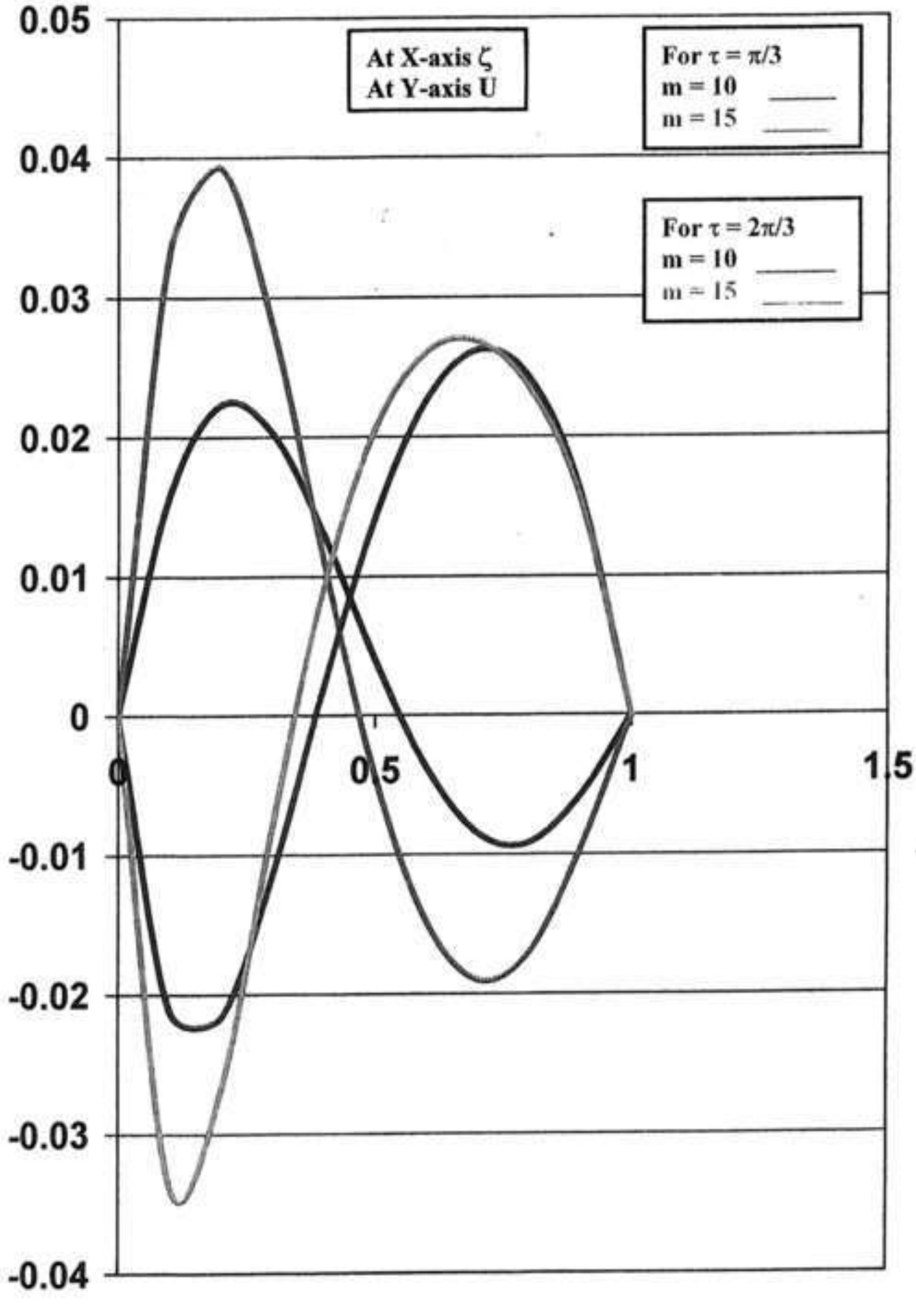
Fig(4) Variation in radial velocity U.



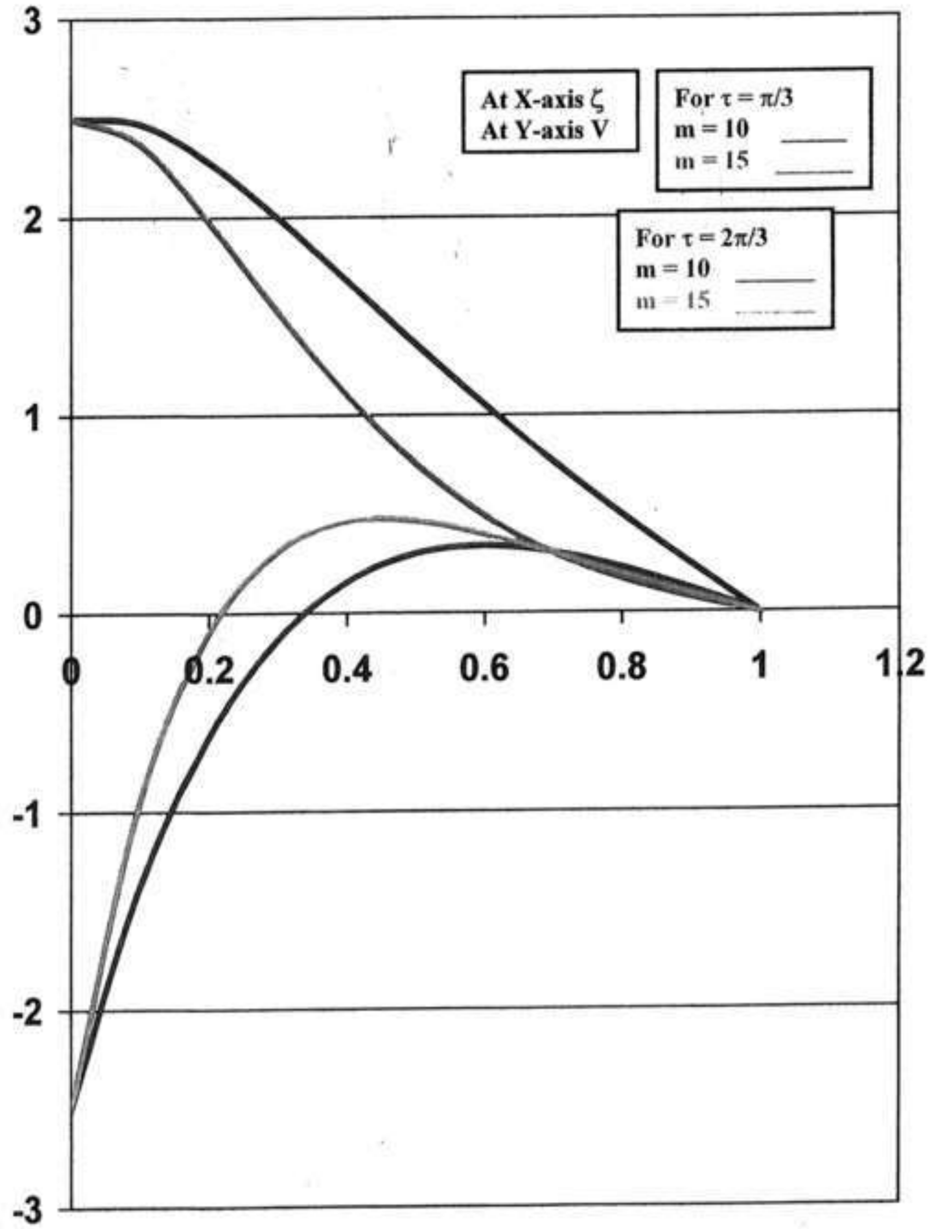
Fig(5) Variation in transverse velocity V.



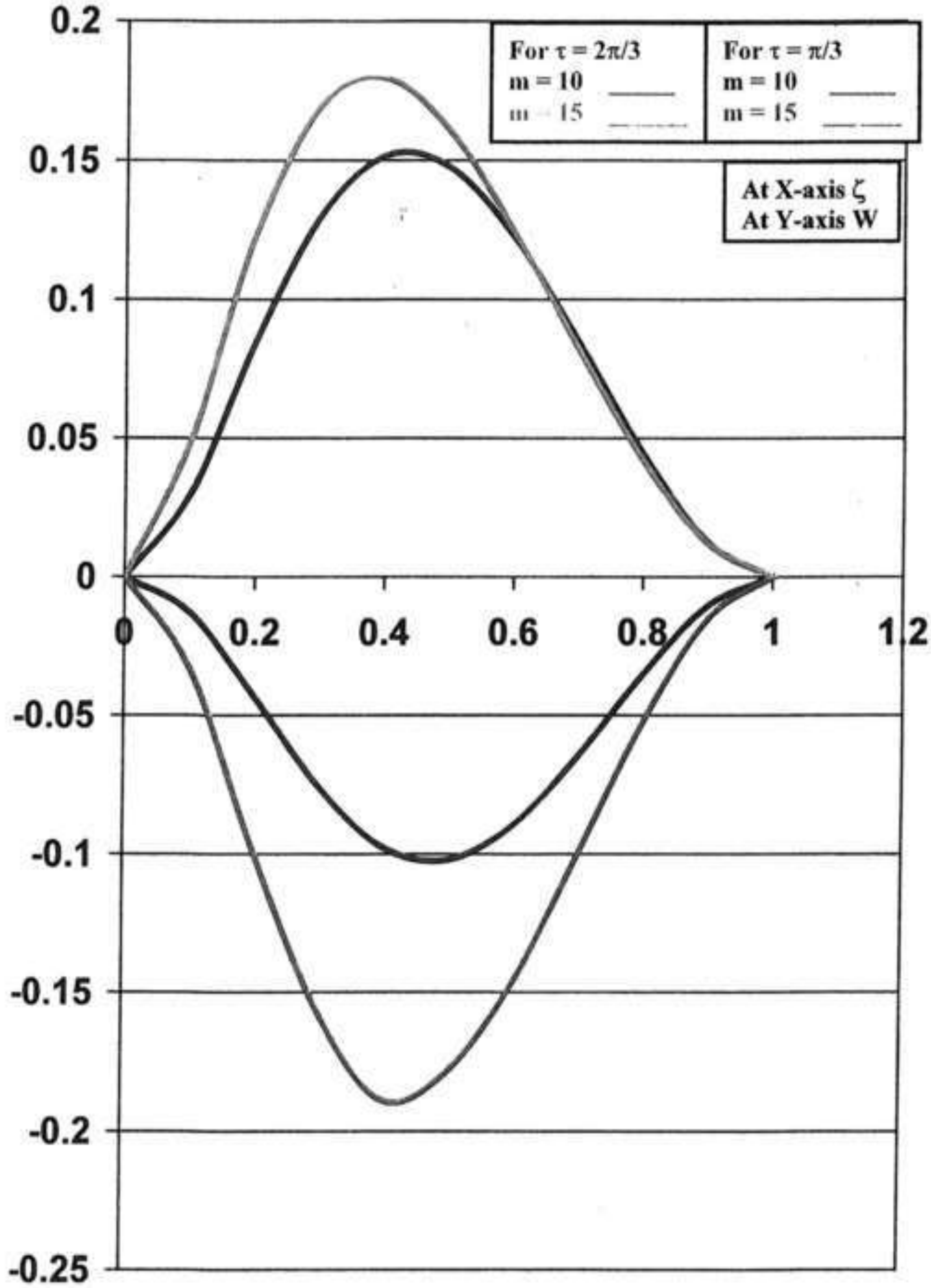
Fig(6) Variation in axial velocity W.



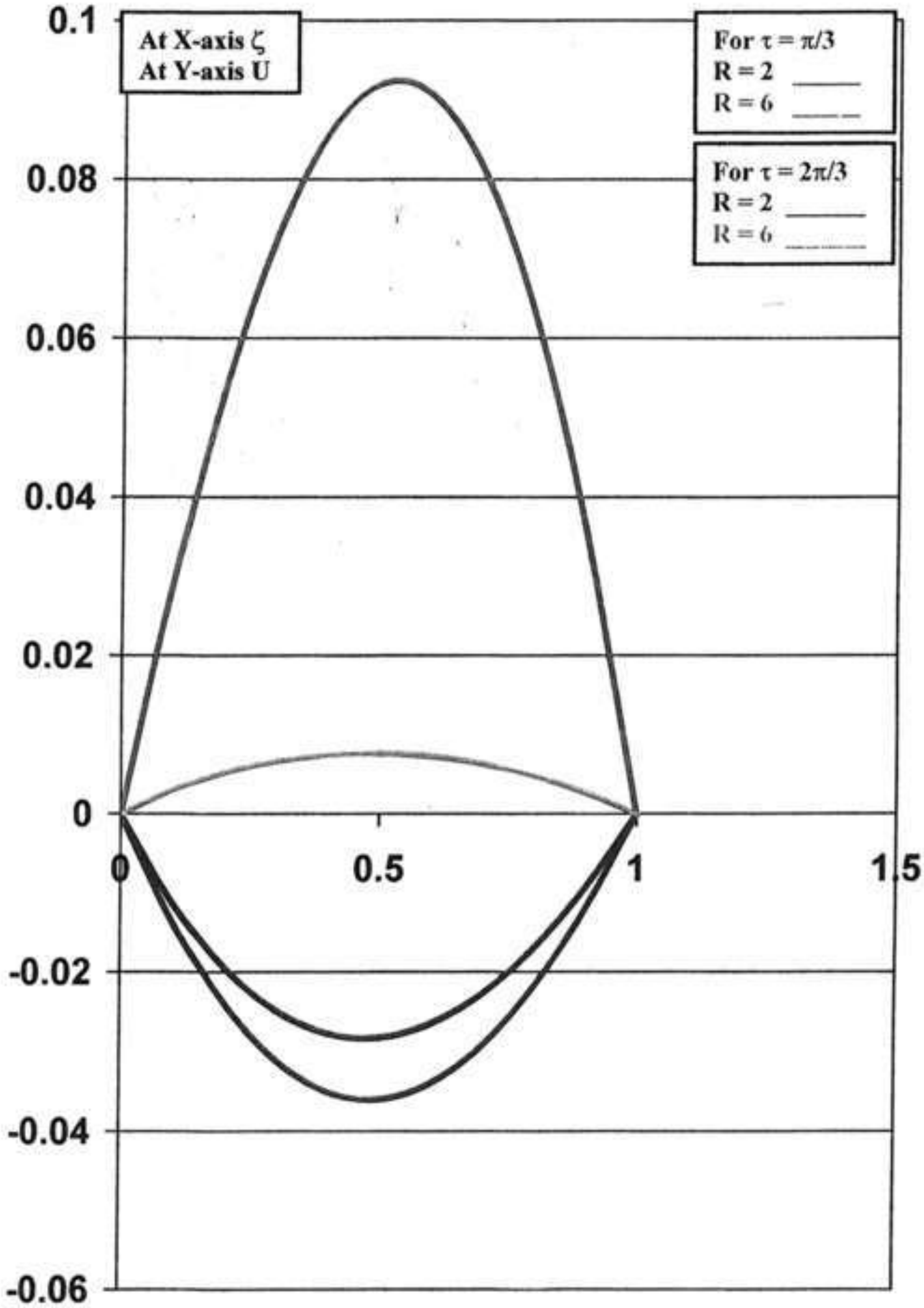
Fig(7) Variation in radial velocity U.



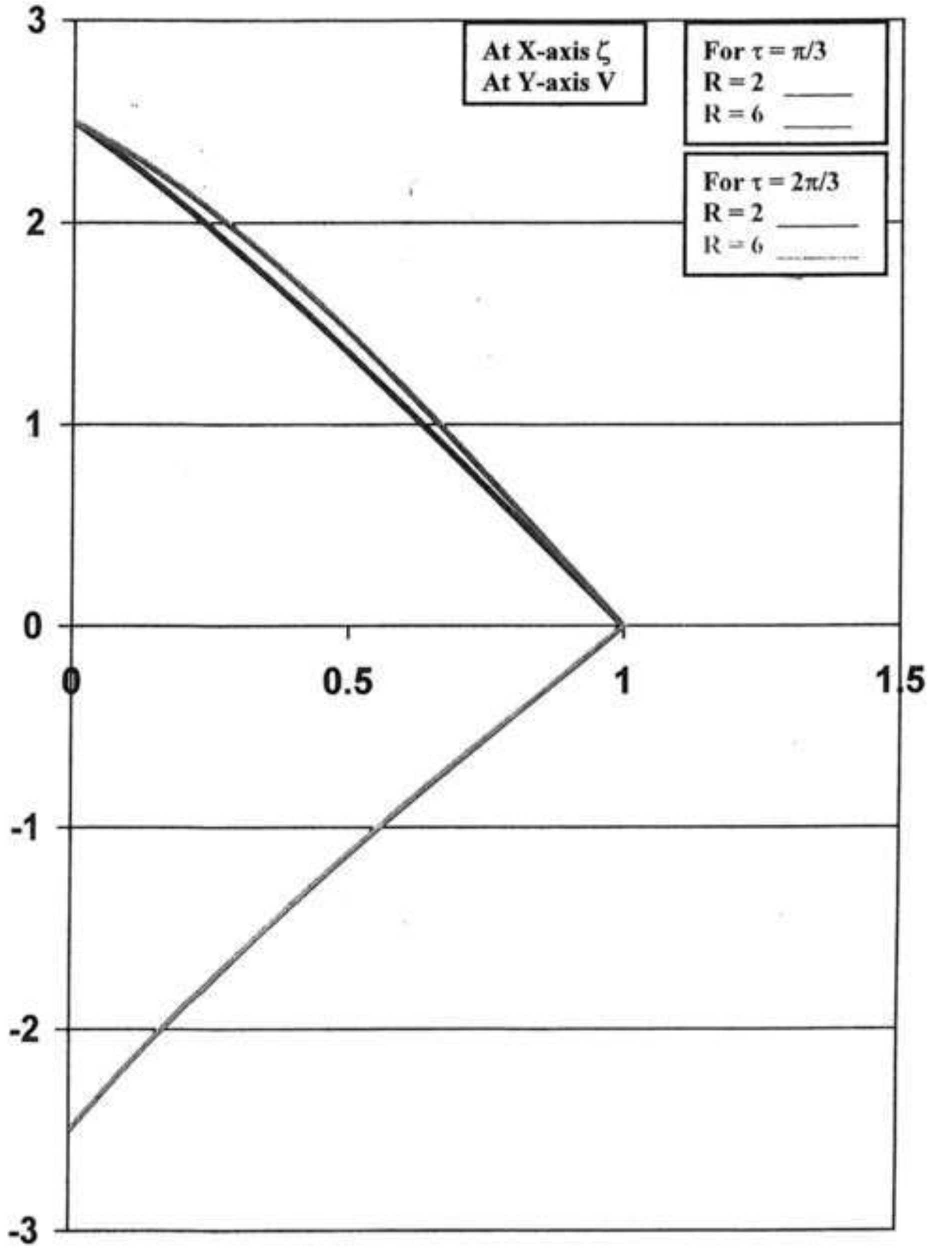
Fig(8) Variation in transverse velocity V.



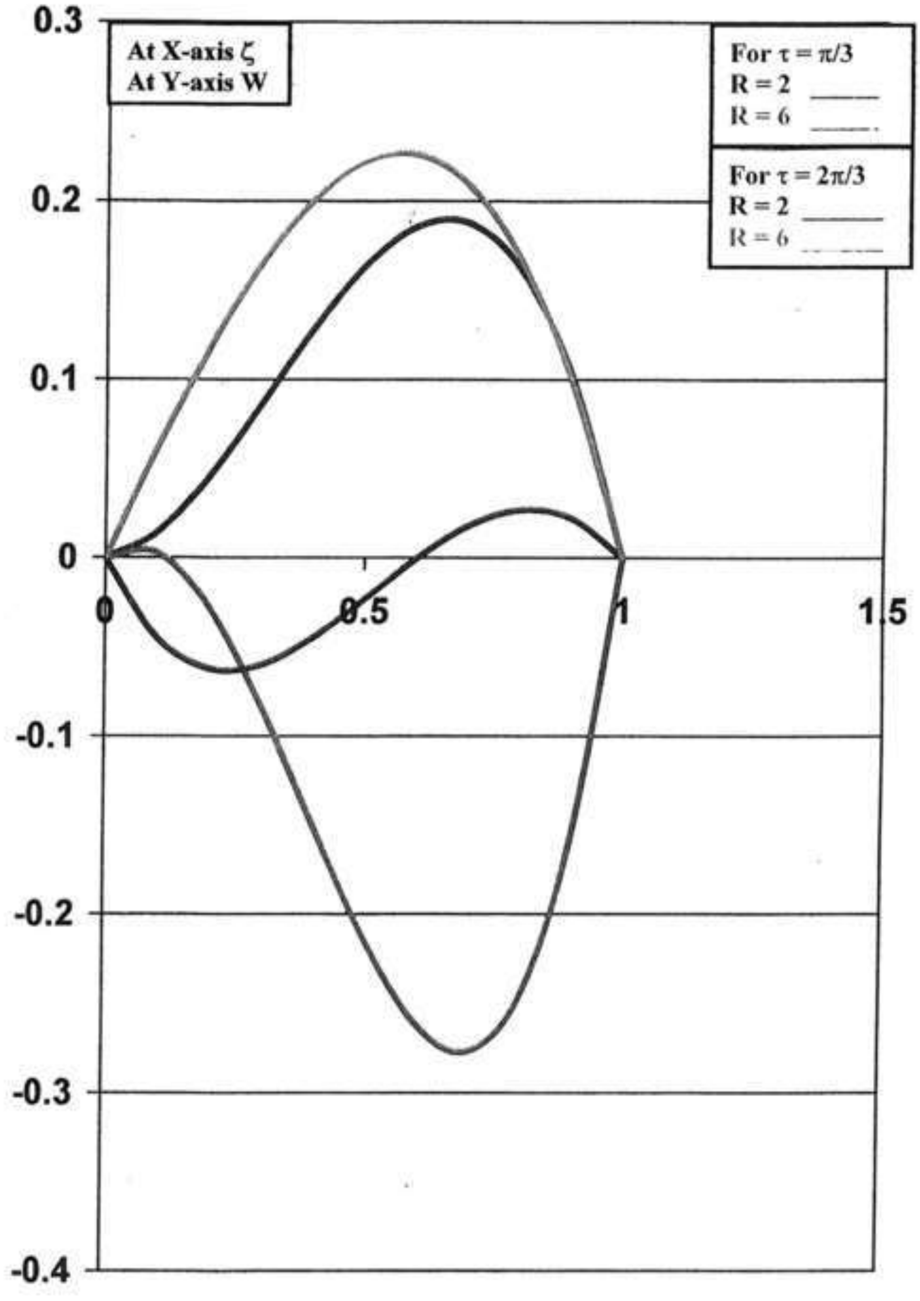
Fig(9) Variation in axial velocity W.



Fig(10) Variation in radial velocity U.



Fig(11) Variation in transverse velocity V.



Fig(12) Variation in axial velocity W.

Chapter No. 4

HEAT TRANSFER IN THE FLOW OF A NON-NEWTONIAN SECOND-ORDER FLUID OVER AN ENCLOSED TORSIONALLY OSCILLATING DISCS WITH UNIFORM SUCTION AND INJECTION IN THE PRESENCE OF THE MAGNETIC FIELD

VI.1 INTRODUCTION

The phenomenon of flow of the fluid over an enclosed torsionally oscillating disc (enclosed in a cylindrical casing) has important engineering applications. The most common practical application of it is the domestic washing machine and blower of curd etc., Soo[64)] has considered first the problem of laminar flow over an enclosed rotating disc in case of Newtonian fluid. Sharma and Agarwal[77)] have discussed the heat transfer from an enclosed rotating disc in case of Newtonian fluid. Thereafter Singh K. R. and H.G. Sharma[78)] have discussed the heat transfer Singh K. R. and H.G. Sharma [78)] have discussed the heat transfer from an enclosed rotating disc in case of Newtonian fluid. Thereafter in the flow of a second-order fluid between two enclosed rotating discs. The torsional oscillations of Newtonian fluids have been discussed by Rosenblat[65)]. He has also discussed the case when the Newtonian fluid is confined between two infinite torsionally oscillating discs[66)]. Sharma & Gupta[67)] have considered a general case of flow of a second-order fluid between two infinite torsionally oscillating discs. Thereafter Sharma & K. R. Singh[79)] have solved the problem of heat transfer in the flow of non-Newtonian second-order fluid between torsionally oscillating plane Riley & Wybrow[71)] have considered the flow induced by the torsional oscillations of an elliptic cylinder. Sadhna kahre[61)] studied the steady flow between a rotating and porous stationary disc in the presence of transverse magnetic field.

Due to complexity of the differential equations and tedious calculations of the solutions, no one has tried to solve the most practical problems of enclosed torsionallly oscillating discs so far. The authors have considered the present problem of heat transfer in the flow of a non-Newtonian second-order fluid over an enclosed torsionally oscillating discs with uniform suction and injection in the presence of the magnetic field and calculated successfully the steady and unsteady part both of the flow and energy functions. The flow and energy functions are expanded in the powers of the amplitude $\in$ (assumed to be small) of the oscillations of the disc. The non-Newtonian effects are exhibited through two dimensionless parameters τ_1 ($=n\mu_2/\mu_1$) and τ_2($=n\mu_3/\mu_1$), where μ_1, μ_2, μ_3 are coefficient of Newtonian viscosity, elastic-viscosity and cross-viscosity respectively, n being the uniform frequency of the oscillation. The variation of temperature distribution with elastic-viscous parameter τ_1, cross –viscous parameter τ_2 (based on the relation $\tau_1 = a\,\tau_2$, where a = -0.2 as for 5.46% poly-iso- butylenes type solution in cetane at 30^0C (Markowiz[38]) Reynolds number R_1 magnetic field m, suction parameter k at different phase difference τ is shown graphically.

VI.2 FORMULATION OF THE PROBLEM

In the three dimensional cylindrical set of co-ordinates (r, θ, z) the system consists of a finite oscillating disc of radius r_s (coinciding with the plane z = 0) performing rotator oscillations of the type rΩCost of small amplitude ϵ , about perpendicular axis r = 0 with a constant angular velocity Ω in an incompressible second-order fluid forming the part of a cylindrical casing or housing. The top of the casing (coinciding with the plane $z = z_0 < r_s$) may be considered as a stationary disc (stator) placed parallel to and at a distance equal to gap length z_0 from the oscillating disc. The symmetrical radial steady outflow has a small mass rate 'm' of radial outflow ('**-m**' for net radial inflow). The inlet condition is taken as a simple radial source flow along z-axis starting from radius r_0. A constant magnetic field B_0 is applied normal to the plane of the oscillating disc. The induced magnetic field is neglected. The lower disc $z = 0$ is maintained at constant temperature T_a while the upper disc $z = z_a$ at constant temperature T_b.

Assuming (u, v, w) as the velocity components along the cylindrical system of axes (r, θ, z) the relevant boundary conditions of the problem are:

$$z = 0, \quad u = 0, \quad v = r\Omega^{i\tau}(\text{Real part}), \quad w = w_0 \quad T = T_a$$

$$z = z_0, \quad u = 0, \quad v = 0, \quad w = w_0 \quad T = T_b$$

(6.1)

where the gap z_0 is assumed small in comparison with the disc radius r_s. The velocity components for the axisymmetric flow compatible with the continuity criterion can be taken as [64,65,66],

$$U = -\xi H'(\zeta,\tau)+(R_m/R_z)\, M'(\zeta,\tau)/\xi,$$

$$V = \xi G'(\zeta,\tau)+(R_l/R_z)\, L(\zeta,\tau)/\xi,$$

$$W = 2H(\zeta,\tau).$$

(6.2)

and for the temperature, we take

$$T = T_b+(v_1\Omega/C_v)\{\phi(\zeta,\tau)+\xi^{2}\Psi(\zeta,\tau)\}$$

(6.3)

where $U = u/\Omega z_0$, $W = w/\Omega z_0$, $\xi = r/\Omega z_0$, ζ,τ are dimensionless quantities and $H(\zeta,\tau)$, $G(\zeta,\tau)$, $L(\zeta,\tau)$, $M'(\zeta,\tau)$, $\phi(\zeta,\tau)$, $\Psi(\zeta,\tau)$ are dimensionless function of the dimensionless variables $\zeta = z/z_0$ and τ = nt. $R_m(=m/2\pi p z_0 v_1)$, R_L ($=L/2\pi\, p z_0 v_1$) are dimensionless number to be called the Reynolds number of net outflow and circulatory flow respectively. R_z ($=\Omega z_0^2/v_1$) be the flow Reynolds number. The small mass rate 'm' of the radial outflow is represented by

$$\mathbf{m} = 2\pi p \int_0^{z_0} r\, u\, dz \qquad (6.4)$$

Using expression (6.2) and (6.3), the boundary conditions (6.1) transform for G, L & H into the following form:

$$G(0,\tau) = \text{Real}(e^{i\tau}),\quad G(1,\tau) = 0,$$

$$L(0,\tau) = 0,\qquad L(1,\tau) = 0,$$

$$H(0,\tau) = k,\qquad H(1,\tau) = 0,$$

$$H'(0,\tau) = 0,\qquad H'(1,\tau) = 0,$$

$$\phi(0,\tau) = 1/E = S,\quad \phi(1,\tau) = 0,$$

$$\Psi(0,\tau) = 0,\qquad \Psi(1,\tau) = 0 \qquad (6.5)$$

where E [$=\Omega v_1/\{C_v(T_a-T_b)\}$] is the Eckert number and k [$=w_0/2\Omega z_0$] is the suction

parameter.

The conditions on M on the boundaries are obtainable form the expression

(6.4) for m as follows:

$M(1, \tau) - M(0, \tau) = 1$

(6.6)

which on choosing the discs as streamlines reduces to

$M(1, \tau) = 1, \qquad M(0, \tau) = 0$

(6.7)

Using eqs. (1.4) and expression (6.2) in equation (1.8) and neglecting the squares

& higher powers of R_m/R_z (assumed small), we have the following equations in dimensionless form:

$-(1/pz_0)(\partial p/\partial \xi)=n\Omega z_0\{\xi\partial H'-(R_m/R_z)(\partial M'/\xi)\}+\Omega^2 z_0\xi(H'^2-2HH''-G^2)+\Omega^2 z_0(R_m/R_z)(2HM''\xi)-\Omega^2 z_0(R_L/R_z)(2LG/\xi)+(v_1\Omega/z_0)\{H'''\xi-(R_m/R_z)(M'''/\xi)\}-(2v_2/z_0)[n\Omega/2]\{(R_m/R_z)(\partial M'''/\xi)-\xi\partial H'''\}+\Omega^2\xi(H''^2-HH^{iv})+(R_m/R_z)(\Omega^2/\xi)(H'''M'+H''M''+H'M'''+HM^{iv})-(R_L/R_z)(2\Omega^2/\xi)(L'G'+LG'')]-(4v_3\Omega^2-z_0)\{(R_m/R_z)(1/2\xi)(H'''M'+H'M'''+H''M'')-(R_L/R_z)(1/2\xi)(2L'G'+LG'')+(\xi/4)(H''^2-G'^2-2H'H''')\}+(\sigma B_0^2\Omega z_0/p)\{-\xi H'+(R_m/R_z)(M'/\xi)\}.$

(6.8)

$0=n\Omega z_0\{\xi\partial G+(R_L/R_z)(\partial L/\xi)\}-(2\Omega^2 z_0\xi)(HG'-H'G)-\Omega^2 z_0(R_m/R_z)(2M'G/\xi)-\Omega^2 z_0(R_L/R_z)(2HL'/\xi)+(v_1\Omega/z_0)\{\xi G''+(R_L/R_z)(L''/\xi)\}+(2v_2/z_0)[(n\Omega/2)\{\xi\partial G''+(R_L/R_z)(\partial L''/\xi)\}+(R_L/R_z)(\Omega^2/\xi)(H''L'+H'''L+HL'''+H'L'')+(\Omega^2/\xi)(HG'''\ H''G')+(R_m/R_z)(2\Omega^2/\xi)(M'G''+M''G')]+(2v_3\Omega^2/z_0)\{\xi(H'G''-H''G')+(R_L+R_z)(1/\xi)(H''L'+H'''L+H'L'')+(R_m+R_z)(1/\xi)(2M''G'+M'G'')-(\sigma B_0^2\Omega z_0/p)\{\xi G+(R_L/R_z)(L/\xi)\}.$

(6.9)

$-(1/pz_0)(\partial p/\partial \zeta)=2n\Omega z_0\partial H+4\Omega^2 z_0 HH'-2v_1\Omega H''/z_0-(2v_2/z_0)$
$\{n\Omega\partial H''+2\Omega^2\xi^2 (H''H'''+G'G'')+ \Omega^2(22H'HH''+2HH''')-$
$(R_m/R_z)2\Omega^2(H''M'''+H'''M'')+ (R_L/R_z)\, 2\Omega^2 (L'G'' + L''G')\}-(2v_3$
$\Omega^2/z_0)\{\xi^2(H''H''' +G'G'')+14H'H''-(R_m/R_z)$
$(H''M'''+H'''M'')+(R_L/R_z)(L'G''+L''G')\}$
(6.10)

$$pC_v(\partial T/\partial t+u\partial T/\partial r+w\partial T/\partial z) = K\{\partial^2 T/\partial r^2+(1/r)\, \partial T/\partial r+\partial^2 T/\partial z^2\}+\Phi$$
(6.11)

where

$$\Phi = \tilde{\tau}^i{}_j d^i{}_j$$
(6.12)

C_v is the specific heat at constant volume, Φ be the viscous-dissipation function, $\tilde{\tau}^i{}_j$ is the mixed deviatoric stress tensor, K is the thermal conductivity, p is the density of the fluid; B and σ are intensity of the magnetic field and conductivity of the fluid considered.

Differentiating (6.8) w.r.t ζ and (6.10) w.r.t ζ and then eliminating $\partial^2 p/\partial\zeta.\partial\xi$ from the equation thus obtained. We get

$-n\Omega z_0\{\xi\partial H''-(R_m/R_z)\partial M''/\xi\}-2\Omega^2 z_0\xi(HH'''-$
$GG')+(R_m/R_z)(2\Omega^2 z_0/\xi)\, (H'M''+HM''')-$
$(R_L/R_z)(2\Omega^2 z_0/\xi)(LG'+L'G)-(v_1\Omega/z_0)\{(R_m/R_z)(M^{iv}/\xi)- \xi H^{iv}\}-(2v_2/z_0)$
$[(n\Omega/2)\{(R_m/R_z)(\partial M^{iv}/\xi)-\xi\partial H^{iv}\}-\Omega^2\xi(2H''H'''+H'H^{iv}+HH^{v}+4G'G'')$
$+(R_m/R_z)(\Omega^2/\xi)(2H'''M''+H^{iv}M'+2H''M'''+2H'M^{iv}+HM^{v})-$
$(R_L/R_z)(2\Omega^{2/}\xi)(2L'G''+L''G'+LG''')]-(2v_3\Omega^2/z_0)\{(R_m+R_z)(1/\xi)$
$(H^{iv}M'+2H'''M''+2H''M'''+H'M^{iv})-$

$(R_L+R_z)(1/\xi)(3L'G''+2L''G'+LG''') - \xi(H'H^{iv}+3G'G''+2H''H''')\}$

$$_0{}^2\Omega z_0/p) - \{\xi H+(R_m/R_z)(M''/\xi)\}=0$$
(6.13)

On equating the coefficients of ξ and $1/\xi$ from the equation (6.9) & (6.13), we get the following equations:

$$G''=R\partial G+2\in R(HG'-H'G)-\tau_1\partial G''-2\in\tau_1(HG'''-H''G')-2\in\tau_2(H'G''H''G')+m^2G$$

(6.14)

$$L''= R\partial L+2\in R(M'G+HL)-\tau_1\partial L''-2\in\tau_1(H''L'+H'''L+HL'''+H'L''+2M'G''+2M''G') -2\in\tau_2(H''L'+H'''L+H'L''+2M''G'+M'G')+m^2L$$
(6.15)

$$H^{iv}=R\partial H+2\in R(HH'''+GG')-\tau_1\partial H^{iv}-2\in\tau_1(H'H^{iv}+HH^{v}+2H''H'''+4G'G'')-2\in\tau_2(H'H^{iv}+2H''H''+3G'G'')+m^2H''$$
(6.16)

$$M^{iv}=R\partial M+2\in R(H'M''+HM'''-LG'-L'G)-\tau_1\partial M^{iv}-2\in\tau_1(2H'''M''+H^{iv}M'+2H''M''' +2H'M^{iv}+HM^{v}-4L'G''-2L''G'-2LG''')-2\in\tau_2(H^{iv}M'+2H'''M''+2H''M'''+ H'M^{iv}-3L'G''-2L''G'-LG''')+m^2M''$$
(6.17)

where $R(=nz_0/v_1)$ is the Reynolds number, $\tau_1(=nv_2/v_1)$, $\tau_2(=nv_3/v_1)$ and $\in$ $(=\Omega/n)$ are the dimensionless parameter, $m^2 = \sigma B_0{}^2 z_0{}^2/\mu_1$ is the dimensionless magnetic field and R_m/R_L = **m/L** $\approx$ 1.

Using (6.3), (6.12) in (6.11) and equating the coefficient of ξ^2, we get

$$\Psi'' = \in RP_r[\partial\Psi/\in - 2H'\Psi + 2H\Psi' - H''^2 + G'^2 - \tau_1(H''\partial H'' + G'\partial G') - 2\in \tau_1 (H'H''^2 + H'G'^2 HH''H''' + HG'G'') - 3\in \tau_2(H'H''^2 + H'G'^2)],$$

$$4\Psi + \phi'' = \in RP_r[\partial\phi/\in + (R_m/R_z)2M'\Psi + 2H\phi' - 12H'^2 + (R_m/R_z)2H''M'' - (R_L/R_z)2L'G' - \tau_1\{12H'\partial H' - (R_m/R_z)(H''\partial M'' + M''\partial H'') + (R_L/R_z)(G'\partial L' + L'\partial L')\} - 2\in \tau_1\{(12H'^3 + 12HH'H'' + (R_m/R_z)(2M'G'^2 - HH'''M'' - H''^2M' - HH''M''') + (R_L/R_z)(HL'G' + 3H'L'G' + 3LG'H'' + HG'L'')\} - \in \tau_2\{(R_m/R_z)(3M'G'^2 - 6H'H''M) + 24H'^3 + 6(R_L/R_z)(H''LG' + H'L'G')\}]$$

Where $P_r = \mu_1 C_v/K$ is the Prandtl number.

For $R_m = R_L = B_0 = 0$, the differential equations (6.8)-(6.10) are identical to those obtained by Sharma & Gupta[67)] (for $S_1 = 1$, $S_2 = 0$), Sharma & Singh[68)] (for $S_1 = 1$, $S_2 = 0$) and for $R_m = R_L = B_0 = 0$, the differential equations (6.8)-(6.10),(6.18), (6.19) are identical to those obtained by Sharma & Singh[79)] (for $S_1 = 1$, $S_2 = 0$).

IV.3 SOLUTION OF THE PROBLEM

Substituting the expressions

$$G(\zeta,\tau) = \sum \in^N G_N(\zeta,\tau)$$

$$L(\zeta,\tau) = \sum \in^N L_N(\zeta,\tau)$$

$$H(\zeta,\tau) = \sum \in^N H_N(\zeta,\tau)$$

$$M(\zeta,\tau) = \sum \in^N M_N(\zeta,\tau)$$

$$\phi(\zeta,\tau) = \sum \in^N \phi_N(\zeta,\tau)$$

$$\Psi(\zeta,\tau) = \sum \in^N \Psi_N(\zeta,\tau)$$

(6.20)

into (6.14) to (6.19) neglecting the terms with coefficient of $\in^2$ (assumed negligible small) and equating the terms independent of $\in$ and coefficient of $\in$, we get the following equations:

$G_0'' = R\,\partial G_0/\partial\tau - \tau_1\,\partial G_0''/\partial\tau + m^2 G_0$

(6.21)

$G_1'' = R\,\partial G_1/\partial\tau - 2R(H_0'G_0' - H_0G_0') - \tau_1\,\partial G_1''/\partial\tau - 2\tau_1(H_0'G_0''' - H_0''G_0') - 2\tau_2(H_0'G_0'' -$

$H_0''G_0') + m^2 G_1$

(6.22)

$L_0'' = R\,\partial L_0/\partial\tau - \tau_1\,\partial L_0''/\partial\tau + m^2 L_0$

(6.23)

$L_1'' = R\,\partial L_1/\partial\tau - 2R(M_0'G_0' - H_0L_0') - \tau_1\,\partial L_1''/\partial\tau - 2\tau_1(H_0'''L_0 + H_0''L_0' + H_0'L_0'' + H_0L_0'''$

$+ 2M_0''G_0' + 2M_0'G_0'') - 2\tau_2(H_0'''L_0 + H_0''L_0' + H_0'L_0'' + 2M_0''G_0' + M_0'G_0'') + m^2 L_1$

(6.24)

$H_0^{iv} = R\partial H_0''/\partial\tau - \tau_1\partial H_0^{iv}/\partial\tau + m^2 H_0''$

(6.25)

$H_1^{iv} = R\,\partial H_1''/\partial\tau + 2R(H_0H_0''' + G_0G_0') - \tau_1\,\partial H_1^{iv}/\partial\tau - 2\tau_1(H_0'H_0^{iv} + H_0H_0^{v} + 2H_0''H_0''' +$

$4G_0'G_0'') - 2\tau_2(3G_0'G_0'' + H_0'H_0^{iv} + 2H_0''H_0''') + m^2 H_1''$

(6.26)

$M_0^{iv} = R\partial M_0''/\partial\tau - \tau_1\partial M_0^{iv}/\partial\tau + m^2 M_0''$

(6.27)

M_1^{iv}=$R\partial M_1''/\partial\tau$-$2R(H_0'M_0''+H_0M_0'''-L_0'G_0-L_0G_0')$- $\tau_1\partial M_1^{iv}/\partial\tau$-$2\tau_1(2H_0'''M_0''$

$+H_0^{iv}M_0'+2H_0''M_0'''-4L_0'G_0''-2L_0''G_0'-2L_0G_0''')+H_0M_0^{v}+2H_0'M_0^{iv})-2\tau_2$ $(2H_0'''M_0''$ $+H_0^{iv}M_0'+2H_0''M_0$ '''$-3L_0'G_0''-2L_0''G_0'-L_0G_0'''+H_0'M_0^{iv})+$ m^2M_1

(6.28)

$\Psi_0''= RP_1\partial\Psi_0$,

(6.29)

$\Psi_1''= RP_r[\partial\Psi_1-2H_0'\Psi_0+2H_0\Psi_0'-H_0''^2-G_0'^2-\tau_1(H_0''\partial H_0''+G_0'\partial G_0')$,

(6.30)

$4\Psi_0+\phi_0''=RP_1\partial\Psi_0$,

(6.31)

$4\Psi_1+\phi_0''=RP_r[\partial\phi_1+(R_m/R_z)2M_0'\Psi_0+2H_0\phi_0'-12H_0'^2+(R_m/R_z)2H_0''M_0''-$ $(R_L/R_z)2L_0'G_0'-\tau_1\{(12H_0'\partial H_0'-(R_m/R_z)(H_0''\partial M_0''+M_0''\partial H_0'')+(R_L/R_z)(G_0'\partial$ $L_0')\}]$. (6.32)

Taking $G_n(\zeta,\tau) = G_{ns}(\zeta)+e^{i\tau}G_{nt}(\zeta)$

$L_n(\zeta,\tau) = L_{ns}(\zeta)+e^{i\tau}L_{nt}(\zeta)$

$H_n(\zeta,\tau) = H_{ns}(\zeta)+e^{2i\tau}H_{nt}(\zeta)$

$M_n(\zeta,\tau) = M_{ns}(\zeta)+e^{2i\tau}M_{nt}(\zeta)$

$\phi_n(\zeta,\tau) = \phi_{ns}(\zeta)+e^{2i\tau}\phi_{nt}(\zeta)$

$\Psi_n(\zeta,\tau) = \Psi_{ns}(\zeta)+e^{2i\tau}\Psi_{nt}(\zeta)$

(6.33)

Complex notation has been adopted here with the convention that only real pats of the complex quantities have the physical meaning.

Using (6.24) and (6.33), the boundary conditions (6.5) & (6.7) for n = 0,1 transform to

$$G_{0s}(0) = 0, \quad G_{0t}(0) = 1, \quad G_{1s}(0) = 0, \quad G_{1t}(0) = 0,$$

$$G_{0s}(1) = 0, \quad G_{0t}(0) = 0, \quad G_{1s}(1) = 0, \quad G_{1t}(1) = 0,$$

$$H_{0s}(0) = k, \quad H_{0t}(0) = 0, \quad H_{1s}(0) = 0, \quad H_{1t}(0) = 0,$$

$$H_{0s}(1) = k, \quad H_{0t}(1) = 0, \quad H_{1s}(1) = 0, \quad H_{1t}(1) = 0,$$

$$H'_{0s}(0) = 0, \quad H'_{0t}(0) = 0, \quad H'_{1s}(0) = 0, \quad H'_{1t}(0) = 0,$$

$$H'_{0s}(1) = 0, \quad H'_{0t}(1) = 0, \quad H'_{1s}(1) = 0, \quad H'_{1t}(1) = 0,$$

$$L_{0s}(0) = 0, \quad L_{0t}(0) = 0, \quad L_{1s}(0) = 0, \quad L_{1t}(0) = 0,$$

$$L_{0s}(1) = 0, \quad L_{0t}(1) = 0, \quad L_{1s}(1) = 0, \quad L_{1t}(1) = 0,$$

$$M'_{0s}(0) = 0, \quad M'_{0t}(0) = 0, \quad M'_{1s}(0) = 0, \quad M'_{1t}(0) = 0,$$

$$M'_{0s}(1) = 0, \quad M'_{0t}(1) = 0, \quad M'_{1s}(1) = 0, \quad M'_{1t}(1) = 0,$$

$$M_{0s}(0) = 0, \quad M_{0t}(0) = 0, \quad M_{1s}(0) = 0, \quad M_{1t}(0) = 0,$$

$$M_{0s}(0) = 0, \quad M_{0t}(0) = 0, \quad M_{1s}(0) = 0, \quad M_{1t}(0) = 0,$$

$$M_{0s}(1) = 0, \quad M_{0t}(1) = 0, \quad M_{1s}(1) = 0, \quad M_{1t}(1) = 0,$$

$$\Psi_{0s}(0) = 0, \quad \Psi_{0t}(0) = 0, \quad \Psi_{1s}(0) = 0, \quad \Psi_{1t}(0) = 0,$$

$$\Psi_{0s}(1) = 0, \quad \Psi_{0t}(1) = 0, \quad \Psi_{1s}(1) = 0, \quad \Psi_{1t}(1) = 0,$$

$$\phi_{0s}(0) = 0, \quad \phi_{0t}(0) = 0, \quad \phi_{1s}(0) = 0, \quad \phi_{1t}(0) = 0,$$

$$\phi_{0s}(1) = 0, \quad \phi_{0t}(1) = 0, \quad \phi_{1s}(1) = 0, \quad \phi_{1t}(1) = 0, \qquad (6.34)$$

Applying (6.33) & (6.34) in eqs. (6.21) to (6.32), we get

$G_{0s}(\zeta) = G1s(\zeta) = 0,$

$G_{0t}(\zeta) = \{1-(e^{f}/2\text{Sinh } f)\}e^{f\zeta}+(e^{f}/2\text{Sinh } f)\, e^{-f\zeta},$

where $f = \{(iR+m^2)/(1+i\tau)\}^{1/2}= A+iB,$

where $A = [[(m^2+R\tau_1)^2+(m^2+R\tau_1)^2+(R-m^2\tau_1)^2\}^{1/2}/\{2(1+\tau_1{}^2\}]^{1/2},$

$B = [[(m^2+R\tau_1)^2+(R-m^2\tau_1)^2\}^{1/2}-(m^2 +R\tau_1)]/\{2(1+\tau_1{}^2\}]^{1/2},$

$G_0(\zeta,\tau) = \text{Real}\{e^{i\tau}G_{0t}(\zeta)\},$

$= (A_3+A_5)\text{Cos}\tau-(A_4+A_6)\text{Sin}\tau,$

where

$A_1=e^{A}(\text{Cos}^2B\text{Sinh}A+ \text{Cosh}A\text{Sin}^2B)/2(\text{Sinh}^2A+\text{Sin}^2B)\},$

$A_2=e^{A}(\text{Sin}B\text{Cos}B\text{Sinh}A-\text{Cos}B\text{Cosh}A\text{Sin}B)/2(\text{Sinh}^2A+\text{Sin}^2B)\},$

$A_3=e^{A\zeta}\{(1-A_1)\text{Cos}B\zeta+A_2\text{Sin}B\zeta\},$

$A_4=e^{A\zeta}\{(1-A_1)\text{Sin}B\zeta+A_2\text{Cos}B\zeta\},$

$A_5=e^{-A\zeta}\{A_1\text{Cos}B\zeta+A_2\text{Sin}B\zeta\},$

$A_6=e^{-A\zeta}\{A_2\text{Cos}B\zeta-A_1\text{Sin}B\zeta\},$

$G_{1\tau}(\zeta) = C_1e^{f\zeta}+ C_2e^{-f\zeta}+Rk[a_1e^{f\zeta}\{\zeta-(1/2f)+ae^{-f\zeta}\{\zeta-(1/2f)\}],$

Where $a_1=1-(e^{f}/2\text{Sinh}f),$

$a_2= e^{f}/2\text{Sinh}f,$

$C_1=-C\{C_2+(Rk/2f)(a_2-a_1)\},$

$C_2=-(Rk/2\text{Sinh}f)(a_1e^{f}+a_2e^{-f}-(a_2\text{Sinh}f/f)\},$

$G(\zeta,\tau)= G_0(\zeta,\tau)+\in G_1(\zeta,\tau)$.

$M_{0t}(\zeta)= M_{1t}(\zeta)=0$,

$M_{0s}(\zeta) = A_1e^{m\zeta}/m^2+ A_2e^{-m\zeta}/m^2+A_3\zeta+A_4$,

$M'_{0s}(\zeta) = A_1e^{m\zeta}/m-A_2e^{-m\zeta}/m^2+A_3$

Where $A_1 = (e^{-m}-1)/ (e^m-1)$,

$A_2 =m^2(e^m-1)/\{(e^m(m-2)-e^{-m}(m+2)\}$,

$A_3 = -(A_1- A_2)/m$,

$A_4 = (A_1+ A_2)/m^2$.

$M_{1s}(\zeta)= C_5e^{m\zeta}/m^2+ C_6e^{-m\zeta}/m^2+(Rk-\tau_1km^2)[C_1e^{m\zeta}(2m\zeta-5)/2m^3+$

$C_5e^{-m\zeta}(2m\zeta+5)/2m^3]+C_7\zeta+C_8$,

where $C_5 = \{m^2(e^{-m}-1)(D_1+D_2-D_3)+m(e^{-m}+m-1)(D_2-D_4\}/4-e^{-m}(m+2)+e^m(m-2)\}$,

$C_6 = \{C_5(e^m-1)-m(D_3-D_4)\}/(e^{-m}-1)$,

$C_7 = -(C_5/m)+(C_6/m)-D_2$,

$C_8 = -(C_5/m^2)-(C_6/m^2)-D_1$

$D_1 = (Rk-\tau_1km^3)(5/2m^3)(C_2-C_1)$,

$D_2 = -(Rk-\tau_1km^2)(3/2m^3)(C_1+C_2)$,

$D_3 = -(Rk-\tau_1km^2)(1/2m^3)\{C_1e^m(2m-5)+ C_2e^{-m}(2m+5)\}$,

$D_4 = (Rk-\tau_1km^2)(1/2m^2)\{C_1e^m(2m-3)- C_2e^{-m}(2m+3)\}$,

$M(\zeta,\tau)= M_0(\zeta,\tau)+\in M_1(\zeta,\tau)$.

$H_{0t}(\zeta)= H_{1t}(\zeta) = 0$,

$H_{0s}(\zeta) = k$

$H_{1s}(\zeta) = C_1 e^{d\zeta}/d^2 + C_2 e^{-d\zeta}/d^2 + C_3\zeta + C_4 + (Z_1 - f^2 Z_2)[\{1-(e^f/2Sinhf\}^2 e^{2f\zeta}/(16fSinh^2 f)]$

where $d = \{2iR+m^2)/(1+2i\tau)\}^{1/2} = C+iD$,

where $C=[[m^2+4R\tau_1)+\{ [m^2+4R\tau_1)^2+[2R-2m^2\tau_1)^2\}^{1/2}]/\{2(1+4\tau_1{}^2)\}]^{1/2}$,

$D=[[m^2+4R\tau_1)^2+\{ [2R-2m^2\tau_1)^2\}^{1/2}-[m^2+4R\tau_1)]/\{2(1+4\tau_1{}^2)\}]^{1/2}$

$Z_1 = 2R/\{(1+i\tau_1)(4f^2-d^2)\}$,

$Z_2 = (8\tau_1+6\tau_2)/\{(1+2i\tau_i)(4f^2-d^2)\}$,

$C_1=\{d(Z_{10}- Z_9-Z_6+Z_5+C_2(e^{-d}-1)\}$,

$C_2=Z_{11}/\{4-e^{-d}(d+2)+e^d(d-2)\}$,

$C_3=-\{(C_1/d)- (C_2/d)+Z_5-Z_6\}$,

$C_4=-\{(C_1/d^2)+(C_2/d^2)+Z_3-Z_4\}$,

$Z_3 = Z_1[\{1-(e^f/2Sinhf)\}^2/4-fe^{2f}/16fSinh^2 f]$,

$Z_4 = Z_2[f\{1-(e^f/2Sinhf)\}^2/4-fe^{2f}/16fSinh^2 f]$,

$Z_5 = Z_1[\{1-(e^f/2Sinhf)\}^2/2+e^{2f}/8Sinh^2 f]$,

$Z_6 = Z_2[f^2\{1-e^f/2Sinhf)\}^2/2+f^2e^{2f}/8Sinh^2 f]$,

$Z_7 = Z_1[e^{2f}\{1-e^f/2Sinhf)\}^2/4f-1/16fSinh^2 f]$,

$Z_8 = Z_2[fe^{2f}\{1-e^f/2Sinhf)\}^2/4-f/16fSinh^2 f]$,

$Z_9 = Z_1[e^{2f}\{1-e^f/2Sinhf)\}^2/2+1/8Sinh^2 f]$,

$Z_{10} = Z_2[f^2e^{2f}\{1-e^f/2Sinhf)\}^2/2-f^2/8fSinh^2 f]$,

$Z_{11} = d^2(e^d-1)(Z_3-Z_4+Z_5-Z_6-Z_7+Z_8)-d(e^d-d-1)\ Z_{10}-Z_9-Z_6+Z_5)$,

$H_1(\zeta,\tau) = \text{Real}\{e^{2i\tau}H_{1t}(\zeta)\}$,

$= (A_{81}+A_{89})\text{Cos}2\tau-(A_{82}+A_{90})\text{Sin}2\tau$,

where

$A_{13} = 4A^2-4B^2-C^2+D^2$,

$A_{14} = 8AB-2CD$,

$A_{15}= 2R(A_{13}-\tau_1A_{14})/\{ (A_{13}-\tau_1A_{14})^2+(\tau_1 A_{13}+ A_{14})^2\}$,

$A_{16}= -2R(A_{13}\tau_1+A_{14})/\{ (A_{13}-\tau_1A_{14})^2+(\tau_1 A_{13}+ A_{14})^2\}$,

$A_{17}= (8\tau_1 +6\tau_2)(A_{13}-2\tau_1A_{14})/\{(A_{13}-2\tau_1A_{14})^2+(2\tau_1A_{13}+ A_{14})^2\}$,

$A_{18}= -(8\tau_1 +6\tau_2)(2\tau_1A_{13}+ A_{14})/\{(A_{13}-2\tau_1A_{14})^2+(2\tau_1A_{13}+ A_{14})^2\}$,

$A_{19}=[A\{(1-A_1)^2-A_2^2\}+2BA_2(1-A_1)]/\{4(A^2+B^2)\}$,

$A_{20}=-[B\{(1-A_1)^2-A_2^2\}+2AA_2(1-A_1)]/\{4(A^2+B^2)\}$,

$A_{21}=\text{Sinh}^2A\text{Cos}^2B-\text{Cosh}^2A\text{Sin}^2B$,

$A_{22}=2\text{Sinh}A\text{Cos}B\text{Cosh}A\text{Sin}B$,

$A_{23}= e^{2A} (A_{21}\text{Cos}2B+A_{22}\text{Sin}2B)/(A_{21}^2+A_{22}^2)$,

$A_{24}= e^{2A} (A_{21}\text{Sin}2B-A_{22}\text{Cos}2B)/(A_{21}^2+A_{22}^2)$,

$A_{25}= (AA_{23}+BA_{24})/\{16(A^2+B^2)\}$,

$A_{26}= (AA_{24}-BA_{23})/\{16(A^2+B^2)\}$,

$A_{27}=A_{15} (A_{19}-A_{25})-A_{16}(A_{20}+A_{26})$,

$A_{28}=A_{16} (A_{19}-A_{25})+A_{15}(A_{20}-A_{26})$,

$B_1=(1-A_1)^2A_2^2$,

$B_2=-2(1-A_1)A_2$,

$X_1 = (A^2-B^2)(A_{17}A_{27}-A_{18}A_{28})-2AB(A_{18}A_{27}+A_{17}A_{28})$,

$X_2 = 2AB(A_{17}A_{27}-A_{18}A_{28})+ (A^2-B^2)(A_{18}A_{27}+A_{17}A_{28})$,

$A_{29}=(X_1A_{15}+X_2A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{30}=(X_2A_{15}-X_1A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{31}=A_{15}\{(B_1/2)+(A_{23}/8)\}-A_{16}\{(B_2/2)+(A_{24}/8)\}$,

$A_{32}=A_{16}\{(B_1/2)+(A_{23}/8)\}-A_{15}\{(B_2/2)+(A_{24}/8)\}$,

$X_3 = (A^2-B^2)(A_{17}A_{31}-A_{18}A_{32})-2AB(A_{18}A_{31}+A_{17}A_{32})$,

$X_4 = 2AB(A_{17}A_{31}-A_{18}A_{32})+ (A^2-B^2)(A_{18}A_{31}+A_{17}A_{32})$,

$A_{33}=(X_3A_{15}+X_4A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{34}=(X_4A_{15}-X_3A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{35}=e^{2A}(A_{19}Cos2B-A_{20}Sin2B)$,

$A_{36}=e^{2A}(A_{20}Cos2B+A_{19}Sin2B)$,

$A_{37}=(AA_{21}-BA_{22})/[16\{(AA_{21}-BA_{22})^2+(BA_{21}-AA_{22})^2\}]$,

$A_{38}=-(BA_{21}+AA_{22})/[16\{(AA_{21}-BA_{22})^2+(BA_{21}-AA_{22})^2\}]$,

$A_{39}=A_{15}(A_{35}-A_{37})-A_{16}(A_{36}-A_{38})$,

$A_{40}=A_{16}(A_{35}-A_{37})-A_{15}(A_{36}-A_{38})$,

$X_5 = (A^2-B^2)(A_{17}A_{39}-A_{18}A_{40})-2AB(A_{18}A_{39}+A_{17}A_{40})$,

$X_6 = 2AB (A_{17}A_{39}-A_{18}A_{40})+ (A^2-B^2) (A_{18}A_{39}+A_{17}A_{40})$,

$A_{41}=(X_5A_{15}+X_6A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{42}=(X_6A_{15}-X_5A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{43}=e^{2A}(B_1Cos2B-B_2Sin2B)/2$,

$A_{44}=e^{2A}(B_2Cos2B+B_1Sin2B)/2$,

$A_{45}=A_{21}/\{8(A_{21}^2+A_{22}^2)\}$,

$A_{46}=-A_{22}/\{8(A_{21}^2+A_{22}^2)\}$,

$A_{47}=A_{15}(A_{43}+A_{45})-A_{16}(A_{44}+A_{46})$,

$A_{48}=A_{16}(A_{43}+A_{45})-A_{15}(A_{44}+A_{46})$,

$X_7 = (A^2-B^2)(A_{17}A_{47}-A_{18}A_{48})-2AB(A_{18}A_{47} +A_{17}A_{48})$,

$X_8 = 2AB (A_{17}A_{47}-A_{18}A_{48})+ (A^2-B^2) (A_{18}A_{47} +A_{17}A_{48})$,

$A_{49}=(X_7A_{15}+X_8A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{50}=(X_8A_{15}-X_7A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{51}=(C^2-D^2)(e^cCosD-1)-2CDe^CSinD$,

$A_{52}=2CD(e^cCosD-1)- (C^2-D^2)e^CSinD$,

$A_{53}=C(e^cCosD-C-1)- D(e^CSinD-D)$,

$A_{54}=D(e^cCosD-C-1)- C(e^CSinD-D)$,

$A_{55}= A_{27}-A_{29}+A_{31}-A_{33}-A_{39}+A_{41}$,

$A_{56}= A_{28}-A_{30}+A_{32}-A_{34}-A_{40}+A_{42}$,

$A_{57}= A_{49}-A_{47}-A_{33}+A_{31}$,

$A_{58}= A_{50}-A_{48}-A_{34}+A_{32}$,

$A_{59}= A_{51}A_{55}-A_{52}A_{56}- A_{53}A_{57}+A_{54}A_{58}$,

$A_{60}= A_{52}A_{55}-A_{51}A_{56}- A_{54}A_{57}+A_{53}A_{58}$,

$A_{65}=4-e^{-C}\{(C+2)CosD+DSinD\}+e^{c}\{(C-2) CosD-DSinD\}$,

$A_{66}=-e^{-C}\{DCosD-(C+2)DSinD\}+e^{c}\{(C-2)SinD+DCosD\}$,

$A_{67}=(A_{59}A_{65}+A_{60}A_{66})/(A_{65}^{2}+A_{66}^{2})$,

$A_{68}=(A_{60}A_{65}-A_{59}A_{66})/(A_{65}^{2}+A_{66}^{2})$,

$A_{69}=CA_{57}-DA_{58}+A_{67}(e^{-c}cosD-1)+ A_{68}e^{-c}SinD$,

$A_{70}=DA_{57}+CA_{58}+A_{67}(e^{-c}cosD-1)+ A_{68}e^{-c}SinD$

$A_{71}=\{A_{69}(e^{c}cosD-1)+A_{70}\ e^{c}SinD\}/\{(e^{c}cosD-1)^{2}+e^{2c}Sin^{2}D\}$,

$A_{72}=\{A_{70}(e^{c}cosD-1)-A_{69}\ e^{c}SinD\}/\{(e^{c}cosD-1)^{2}+e^{2c}Sin^{2}D\}$,

$A_{73}=-[\{(CA_{71}+DA_{72}-CA_{67}-DA_{68})/(C^{2}+D^{2})\}+A_{31}-A_{33}]$,

$A_{74}=-[\{(CA_{72}-DA_{71}-CA_{68}-DA_{67})/(C^{2}+D^{2})\}+A_{32}-A_{3}]$,

$A_{75}=\{A_{71}(C^{2}-D^{2})+2CDA_{72}\}/\{(C^{2}-D^{2})+4C^{2}D^{2}\}$,

$A_{76}=\{A_{72}(C^{2}-D^{2})-2CDA_{71}\}/\{(C^{2}-D^{2})+4C^{2}D^{2}\}$,

$A_{77}=\{A_{67}(C^{2}-D^{2})+2CDA_{68}\}/\{(C^{2}-D^{2})+4C^{2}D^{2}\}$,

$A_{78}=\{A_{68}(C^{2}-D^{2})+2CDA_{67}\}/\{(C^{2}-D^{2})+4C^{2}D^{2}\}$,

$A_{79}=-(A_{75}+A_{77}+A_{27}-A_{29})$,

$A_{80}=-(A_{76}+A_{78}+A_{28}-A_{30})$,

$A_{81}=e^{C\zeta}(A_{75}CosD\zeta-A_{76}SinD\zeta)+ e^{-C\zeta}(A_{77}CosD\zeta+A_{78}SinD\zeta)+A_{73}\zeta+A_{79}$,

$A_{82}=e^{C\zeta}(A_{76}CosD\zeta+A_{75}SinD\zeta)+ e^{-C\zeta}(A_{78}CosD\zeta-A_{77}SinD\zeta)+A_{74}\zeta+A_{80}$,

$A_{83}= A_{15}-(A^{2}-B^{2})A_{17}+2ABA_{18}$,

$A_{84}= A_{16}-(A^2-B^2)A_{18}-2ABA_{17}$,

$A_{85}=e^{2A\zeta}(A_{19}Cos2B\zeta-A_{20}Sin2B\zeta)$,

$A_{86}=e^{2A\zeta}(A_{20}Cos2B\zeta+A_{19}Sin2B\zeta)$,

$A_{87}=e^{2A(1-\zeta)}(A_{37}Cos2B(1-\zeta)-A_{38}Sin2B(1-\zeta)\}$,

$A_{88}=e^{2A(1-\zeta)}(A_{38}Cos2B(1-\zeta)+A_{37}Sin2B(1-\zeta)\}$,

$A_{89}=A_{83}(A_{85}-A_{87})-A_{84}(A_{86}-A_{88})$,

$A_{90}=A_{84}(A_{85}-A_{87})+A_{83}(A_{86}-A_{88})$,

$H(\zeta,\tau)=H_0(\zeta,\tau)+\in H_1(\zeta,\tau)= \in H_1(\zeta,\tau)$

$L_{0s}(\zeta)=L_{0t}(\zeta)= L_{1s}(\zeta) = 0$

$L_{1t}(\zeta) = C_5e^{f\zeta}+ C_6e^{-f\zeta}+\{a_3e^{(m+f)\zeta}+ a_7e^{-(m+f)\zeta}\}/(m^2+2mf)+ \{a_4e^{-(m+f)\zeta}+ a_6e^{(m+f)\zeta}\}/ (m^2-2mf)+(\zeta/2f) \{a_5e^{f\zeta}+ a_8e^{-f\zeta})- \{a_5e^{f\zeta}+ a_8e^{-f\zeta})/4f^2$,

Where

$C_5=-[C_6+\{(a_{3+}a_7)/m^2+2mf)\}+\{(a_4+a_6)/(m^2-2mf)\}-\{(a_5+a_8)/(4f^2)\}]$,

$C_6=[[a_3\{e^{(m+f)}-e^f\}+\{a_7e^{-(m+f)}-e^f\}]/(m^2+2mf)+[a_4\{e^{-(m-f)}-e^f\}+a_6\{e^{(m-f)}-e^f\}](m^2-2mf)+(a_5e^f-a_8e^{-f})/2f+a_8Sinhf/2f^2]/(2Sinhf)$,

$a_1=1-(e^f/2Sinhf)$,

$a_2=e^f/2Sinhf$,

$a_3=\{2RC_1a_1/m(1+i\tau_1)\}-\{4(\tau_1+\tau_2)C_1a_1f/(1+i\tau_1\})-\{(4\tau_1+2\tau_2)C_1a_1f^2/m(1+i\tau_1\})$,

$a_4=-\{2RC_2a_1/m(1+i\tau_1)\}-\{4(\tau_1+\tau_2)C_2a_1f/(1+i\tau_1\})-\{(4\tau_1+2\tau_2)C_2a_1f^2/m(1+i\tau_1\})$,

$a_5=\{2RC_3a_1/(1+i\tau_1)\}-\{4\tau_1+2\tau_2)C_2a_1f^2/(1+i\tau_1),$

$a_6=\{2RC_1a_2/m(1+i\tau_1)\}-\{4(\tau_1+\tau_2)C_1a_2f/(1+i\tau_1\})-$
$\{(4\tau_1+2\tau_2)C_1a_2f^2/m(1+i\tau_1\}),$

$a_7=-\{2RC_2a_2/m(1+i\tau_1)\}-\{4(\tau_1+\tau_2)C_2a_2f/(1+i\tau_1\})-$
$\{(4\tau_1+2\tau_2)C_2a_2f^2/m(1+i\tau_1\}),$

$a_8=\{2RC_3a_2/(1+i\tau_1)\}-\{4\tau_1+2\tau_2)C_3a_2f^2/(1+i\tau_1\}),$

$L_1(\zeta,\tau)=\text{Real}\{e^{i\tau}L_{1t}(\zeta)\},$

$= (B_{47}+B_{51}+B_{55}+B_{59}-B_{63})\text{Cos}\tau-(B_{48}+B_{52}+B_{56}+B_{60}-B_{64})\text{Sin}\tau,$

$B_3=1/(1+\tau_1^{\,2}),$

$B_4=-\tau_1/(1+\tau_1^{\,2}),$

$B_5=(2RC_1/m)\{B_3(1-A_1)+B_4A_2\}-4(\tau_1+\tau_2)C_1\{(1-A_1)(AB_3-$
$BB_4)+A_2(AB_4+BB_3)\}-(4\tau_1+2\tau_2)C_1/m\}(A^2-B^2)\{B_3(1-A_1)+B_4A_2\}-$
$2AB\{B_4(1-A_1)- B_3A_2\}],$

$B_6=(2RC_1/m)\{B_4(1-A_1)-B_3A_2\}-4(\tau_1+\tau_2)C_1\{(1-A_1)(AB_4-BB_3)+A_2(AB_3-$
$BB_4)\}-(4\tau_1+2\tau_2)C_1/m\}[2AB\{B_3(1-A_1)+B_4A_2\}+(A^2-B^2)\{B_4(1-A_1)-$
$B_3A_2\}],$

$B_7=-(2RC_2/m)\{B_3(1-A_1)+B_4A_2\}-4(\tau_1+\tau_2)C_2\{(1-A_1)(AB_3-$
$BB_4)+A_2(AB_4+BB_3)\}+ (4\tau_1+2\tau_2)C_2/m\}(A^2-B^2)\{B_3(1-A_1)+B_4A_2\}-$
$2AB\{B_4(1-A_1)- B_3A_2\}],$

$B_8=-(2RC_2/m)\{B_4(1-A_1)-B_3A_2\}-4(\tau_1+\tau_2)C_2\{(1-A_1)(AB_4+BB_3)+A_2(AB_3-$
$BB_4)\}+ \{(4\tau_1+2\tau_2)C_2/m\}[2AB\{B_3(1-A_1)+B_4A_2\}+(A^2-B^2)\{B_4(1-A_1)-$
$B_3A_2\}],$

B_9=2RC_3{B_3(1-A_1)+B_4A_2}-4(τ_1+2τ_2)C_3[(A^2-B^2){B_3(1-A_1)+B_4A_2}-(2AB){B_4(1-A_1)-

B_3A_2}],

B_{10}=2RC_3{B_4(1-A_1)+B_3A_2}-4(τ_1+2τ_2)C_3[2AB){B_3(1-A_1)+B_4A_2}+(A^2-B^2){B_4(1-A_1)-

B_3A_2}],

B_{11}=(2RC_1/m){B_3A_1-B_4A_2}+4(τ_1+τ_2)C_1{A{B_3A_1-B_4A_2}-B(B_4A_1+B_3A_2)}-{(4τ_1+2τ_2)C_1/m}[{B_3A_1-B_4A_2)(A^2-B^2)-2AB{B_4A_1+B_3A_2}],

B_{12}=(2RC_1/m){B_4A_1+B_3A_2}+4(τ_1+τ_2)C_1{B{B_3A_1-B_4A_2}+A(B_4A_1+B_3A_2)}- {(4τ_1+2τ_2)C_1/m}[{B_3A_1-B_4A_2)2AB+(A^2-B^2)(B_4A_1+B_3A_2}],

B_{13}=-(2RC_2/m){B_3A_1-B_4A_2}+4(τ_1+τ_2)C_2{A{B_3A_1-B_4A_2}-B(B_4A_1+B_3A_2)}+ {(4τ_1+2τ_2)C_2/m}[{B_3A_1-B_4A_2)(A^2-B^2)-2AB{B_4A_1+B_3A_2}],

B_{14}=-(2RC_2/m){B_4A_1+B_3A_2}+4(τ_1+τ_2)C_2{B{B_3A_1-B_4A_2}+A(B_4A_1+B_3A_2)}- {(4τ_1+2τ_2)C_2/m}[{B_3A_1-B_4A_2)2AB+(A^2-B^2)(B_4A_1+B_3A_2}],

B_{15}=2RC_3{B_3A_1-B_4A_2}-4(τ_1+τ_2)C_3{(A^2-B^2)(B_3A_1-B_4A_2}-2AB(B_4A_1+B_3A_2)],

B_{16}=2RC_3{B_4A_1+B_3A_2}-4(τ_1+τ_2)C_3{(B_3A_1-B_4A_2}2AB+(A^2-B^2)(B_4A_1+B_3A_2)],

Y_1=B_5CosB($e^{(m+A)}$-e^A)-B_6SinB($e^{(m+A)}$-e^A),

Y_2=B_6CosB($e^{(m+A)}$-e^A)+B_5SinB($e^{(m+A)}$-e^A),

$Y_3=B_{13}CosB(e^{-(m+A)}-e^A)+B_{14}SinB(e^{-(m+A)}+e^A)$,

$Y_4=B_{14}CosB(e^{-(m+A)}-e^A)-B_{13}SinB(e^{-(m+A)}+e^A)$,

$B_{17}=(Y_1+Y_3)$,

$B_{18}=(Y_2+Y_4)$,

$Y_5=(e^{-(m-A)}-e^A)(B_7CosB- B_8SinB)$,

$Y_6=(e^{-(m-A)}-e^A)(B_8CosB- B_7SinB)$,

$Y_7= B_{11}CosB(e^{(m-A)}-e^A)+ B_{12}SinB(e^{(m-A)}+e^A)$

$Y_8= B_{12}CosB (e^{(m-A)}-e^A)-B_{11}SinB(e^{(m-A)}+e^A)$,

$B_{19}=(Y_5+Y_7)$,

$B_{20}=(Y_6+Y_8)$,

$B_{21}=(m^2+2mA)/\{(m^2+2mA)^2+4m^2B^2\}$,

$B_{22}=-2mB/\{(m^2+2mA)^2+4m^2B^2\}$,

$B_{23}=(m^2-2mA)/\{(m^2-2mA)^2+4m^2B^2\}$,

$B_{24}=2mB/\{(m^2+2mA)^2+4m^2B^2\}$,

$B_{25}=e^A (B_9CosB- B_{10}SinB)- e^{-A} (B_{15}CosB+B_{16}SinB)$,

$B_{26}=e^A (B_{10}CosB+B_9SinB)- e^{-A} (B_{16}CosB-B_{15}SinB)$,

$B_{27}=(AB_{25}+BB_{26})/\{2(A^2+B^2)\}$,

$B_{28}=(AB_{26}-BB_{25})/\{2(A^2+B^2)\}$,

$B_{29}=B_{15}SinhACosB-B_{16}CosASinhB$,

$B_{30}=B_{16}SinhACosB+B_{15}CosASinhB$,

$B_{31}=\{B_{29}(A^2-B^2)+2B_{30}AB\}/[2\{(A^2-B^2)^2+4A^2B^2\}],$

$B_{32}=\{B_{30}(A^2-B^2)-2B_{29}AB\}/[2\{(A^2-B^2)^2+4A^2B^2\}],$

$B_{33}=B_{17}B_{21}- B_{18}B_{22},$

$B_{34}=B_{18}B_{21}+ B_{17}B_{22},$

$B_{35}=B_{19}B_{23}-B_{20}B_{24},$

$B_{36}=B_{20}B_{23}+B_{19}B_{24},$

$B_{37}=\{(B_{33}+B_{35}+B_{27}+B_{31})SinhACosB+(B_{34}+B_{36}+B_{28}+B_{32})CosASinhB\}/\{2($

$Sinh^2ACos^2B+ Cos^2A\ Sinh^2B)\},$

$B_{38}=\{(B_{34}+B_{36}+B_{28}+B_{32})\ SinhACosB-(B_{33}+B_{35}+B_{27}+B_{31})\ CosASinhB\}/\{2(Sinh^2ACos^2B+ Cos^2A\ Sinh^2B)\},$

$B_{39}=B_{21}(B_5+B_{13})-B_{22}(B_6+B_{14}),$

$B_{40}=B_{22}(B_5+B_{13})+B_{21}(B_6+B_{14}),$

$B_{41}=B_{23}(B_7+B_{11})-B_{24}(B_8+B_{12}),$

$B_{42}=B_{24}(B_7+B_{11})+B_{23}(B_8+B_{12}),$

VI.4 RESULTS AND DISCUSSION

The variation of the temperature distribution with ζ at R = 7, P = 6, ζ = 5, and ϵ =0.02, k =15,m = 10, E = 5 for different values of τ_1= 1, 1.2, 3 when $\tau = \pi/3$ and $2\pi/3$ is shown in fig (1) and fig (2) respectively. From fig (1) and fig (2), the graph of the temperature variation is parabolic with vertex downwards. It is also clear from these figures that the temperature is minimum at the middle of the gap-length and remains negative throughout the gap-length except near the surface of the lower disc. It is seen from fig (1) that temperature increases with an increase in elastic-viscous parameter τ in the first half and being overlapped in the second half of the gap-length. It is observed from fig(2) that the temperature decreases with an increase in τ_1 in the middle of the gap-length and is being overlapped thereafter.

The variation of the temperature distribution with ζ at $\tau_1 = 5$, P = 6, $\zeta = 5$, $\epsilon = 0.02$, k = 15, m = 10, E = 5 for different values of R = 1, 1.5, 2 when $\tau = \pi/3$ and $2\pi/3$ is shown in fig (3) and fig (4) respectively. From fig (3) and fig (4), the graph of the temperature variation is parabolic with vertex downwards. It is also evident from these figures that the temperature is minimum at the middle of the gap length and remains negative throughout the gap-length except near the surface of the lower disc. It is also. It is also clear from these that temperature decreases with an increase in Reynolds number R throughout the gap-length.

The variation of the temperature distribution with ζ at R = 7, $\tau_1 = 5$, P = 6, $\zeta = 5$, $\epsilon = 0.02$, k = 15, E = 5 for different values of m = 1, 10, 20 when $\tau = \pi/3$ and $2\pi/3$ is shown in fig (5) and fig (6) respectively. From fig (5) and fig (6), the graph of the temperature variation is parabolic with vertex downwards. It is also observed from these figures that the temperature is minimum at the middle of the gap-length and remains negative throughout the gap-length except near the surface of the lower disc. It is also seen from fig (5) that the temperature increases with an increase in magnetic field parameter m and start overlapping near the upper disc and from fig (6), temperature decreases with an increase in magnetic field parameter m n the first half and being overlapped in the second half of the gap-length.

The variation of the temperature distribution with ζ at R = 7, τ_1= 5, P = 6, ζ = 5, ϵ =0.02, k =15, E = 5 for different values of Suciton parameter k = 1, 2, 3 when $\tau = \pi/3$ and $2\pi/3$ is shown in fig (7) and fig (8) respectively. It is observed from fig (7) and fig (8), that the temperature decreases, attains its minimum value at the middle of the gap length approximately and increases thereafter upto the surface of the upper disc for all values of suction parameter k. It is alos evident from these figure that temperature decreases with an increase in suction parameter k throughout the gap-length.

The variation of the nusselt number Nu_a with ξat m = 10, R = 7, τ_1= 5, P = 6, ξ_0 = 5, $\in$ =0.02, E = 5, τ=$\pi/3$ for different values of Suciton parameter k = 1, 3, 7 is shown in fig (9). It is seen from this figure that the Nusselt number decreases with an increase in ξ throughout the gap-length. It is also observed from this figure that Nusselt number decreases with an increase in suction parameter k throughout the gap-length.

The variation of the Nusselt number Nu_b with ξ at m = 10, R = 7, τ_1= 5, P = 6, ξ_0 = 5, $\in$ =0.02, E = 5, τ= $\pi/3$ for different values of suction parameter k = 1, 3, 7 is shown in fig (10). It is clear from this figure that the Nusselt number increases with an increase in ξ throughout the gap-length. It is also evident from this figure that Nusselt number increases with an increase in suction parameter k throughout the gap-length.

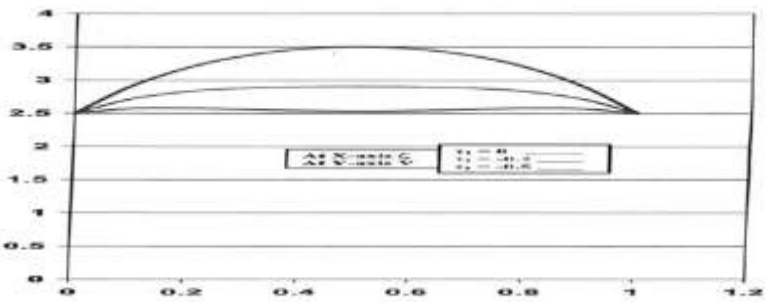


Fig (3) Variation of transverse velocity at different elastico -viscous parameter at $\tau = \pi/3$

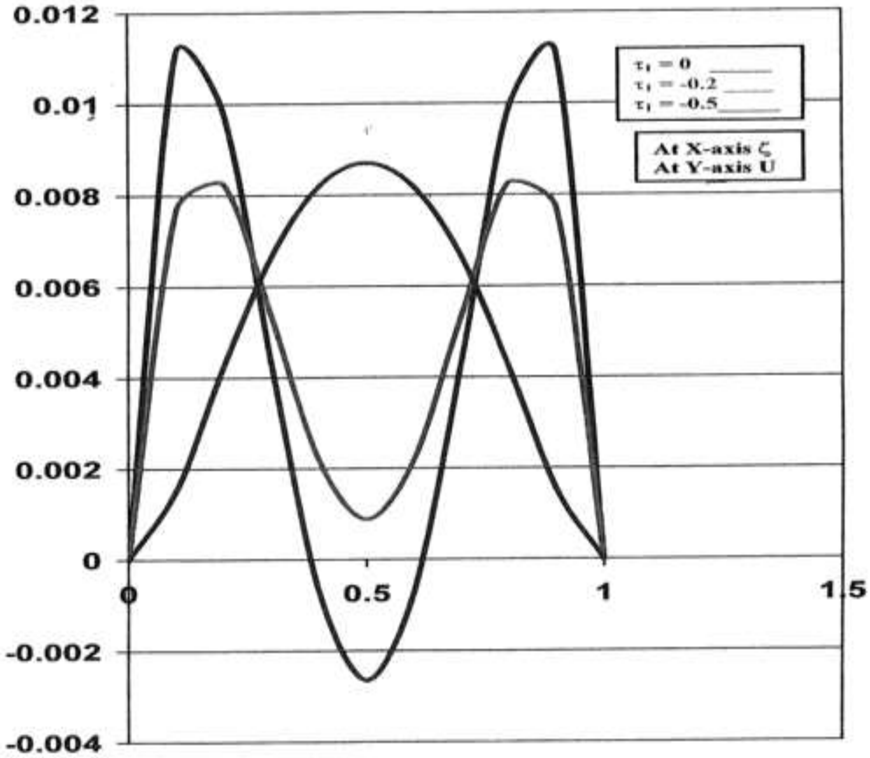


Fig (1) Variation of radial velocity at different elastico-viscous parameter at $\tau = \pi/3$

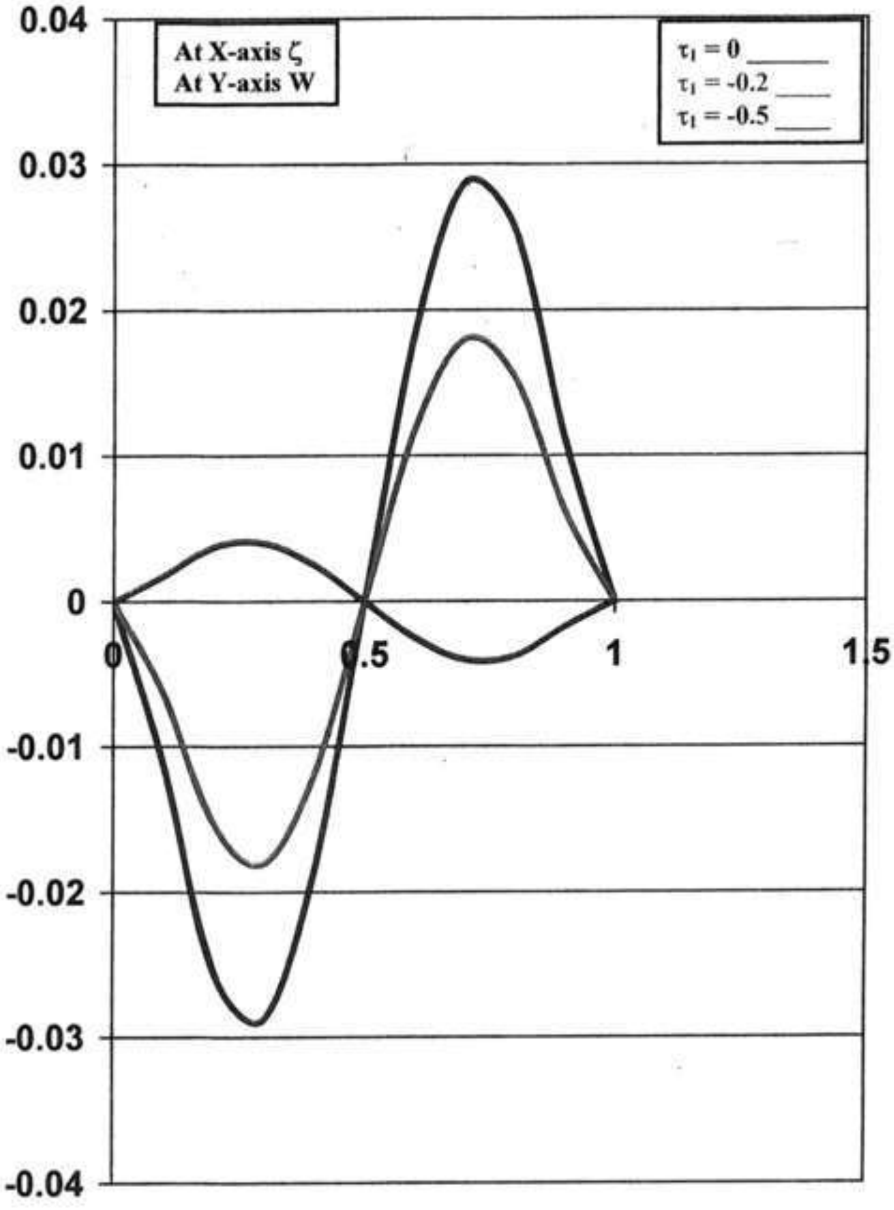


Fig (3) Variation of axial velocity at different elastico -viscous parameter at $\tau = \pi/3$

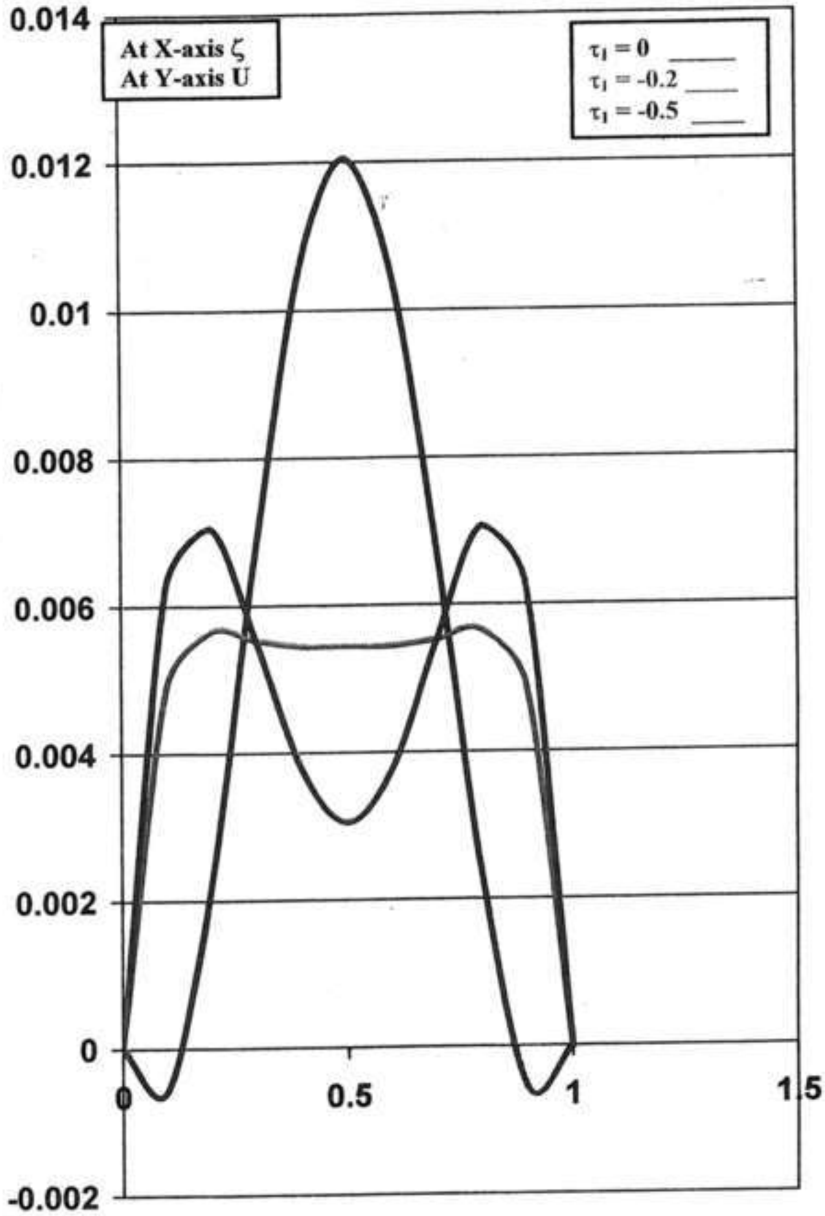


Fig (4) Variation of radial velocity at different elastico-viscous parameter at $\tau = 0$.

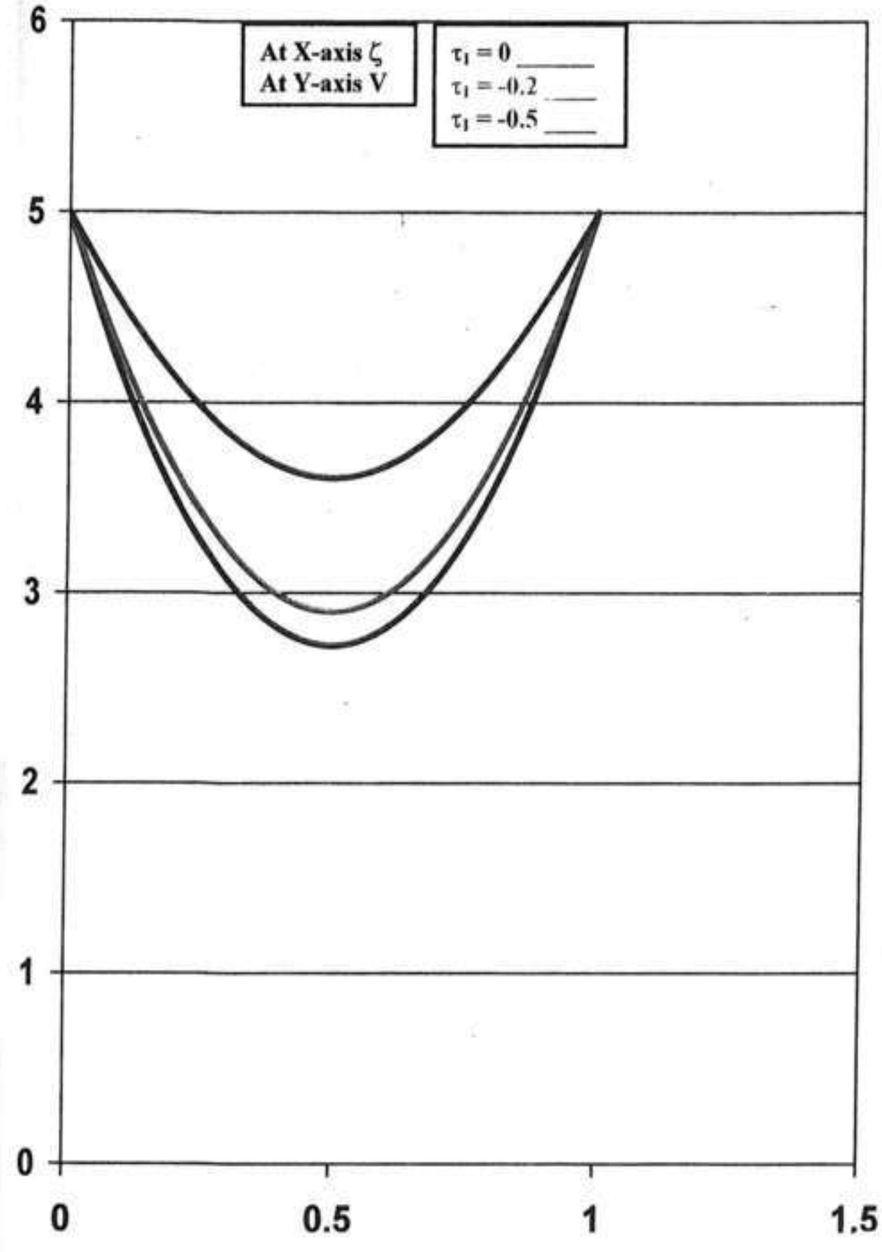


Fig (5) Variation of transverse velocity at different elastico -viscous parameter at $\tau = 0$.

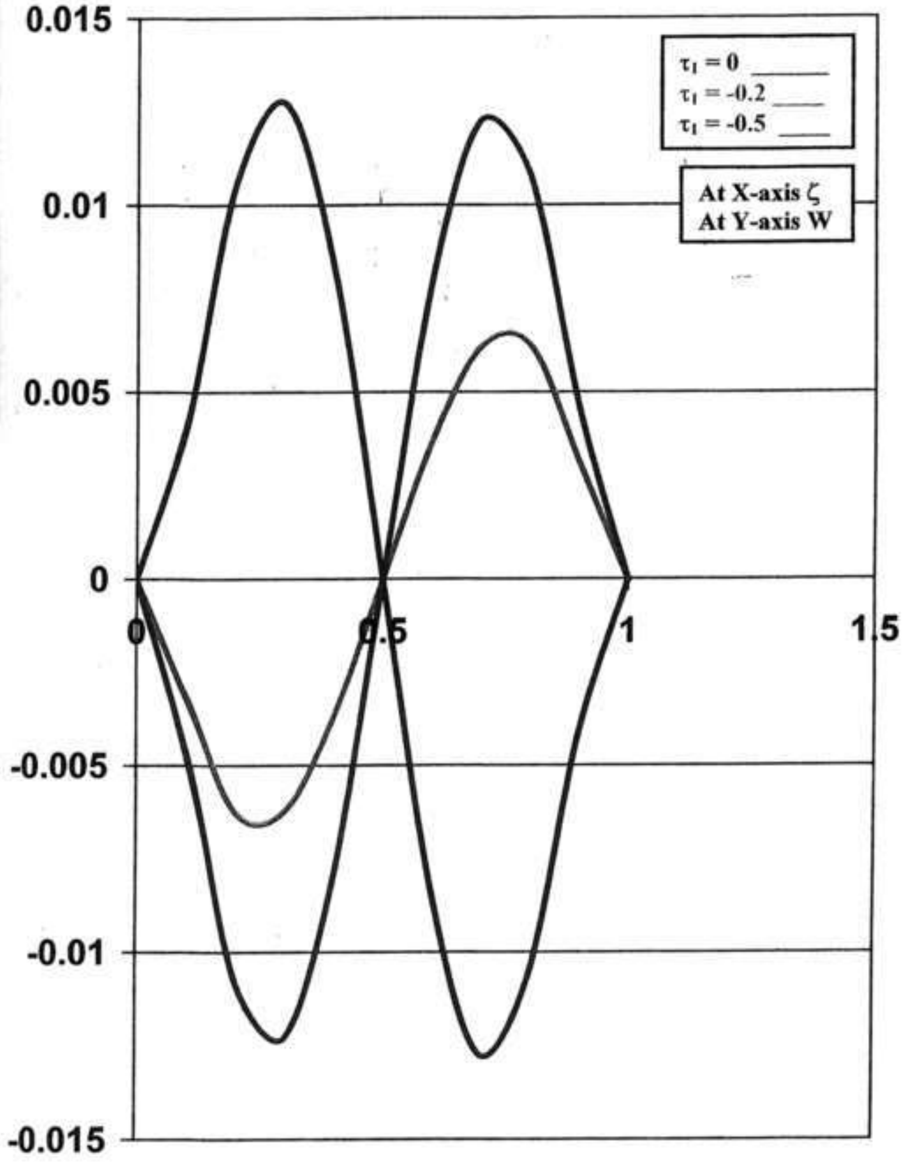


Fig (6) Variation of axial velocity at different elastico -viscous parameter at $\tau = 0$

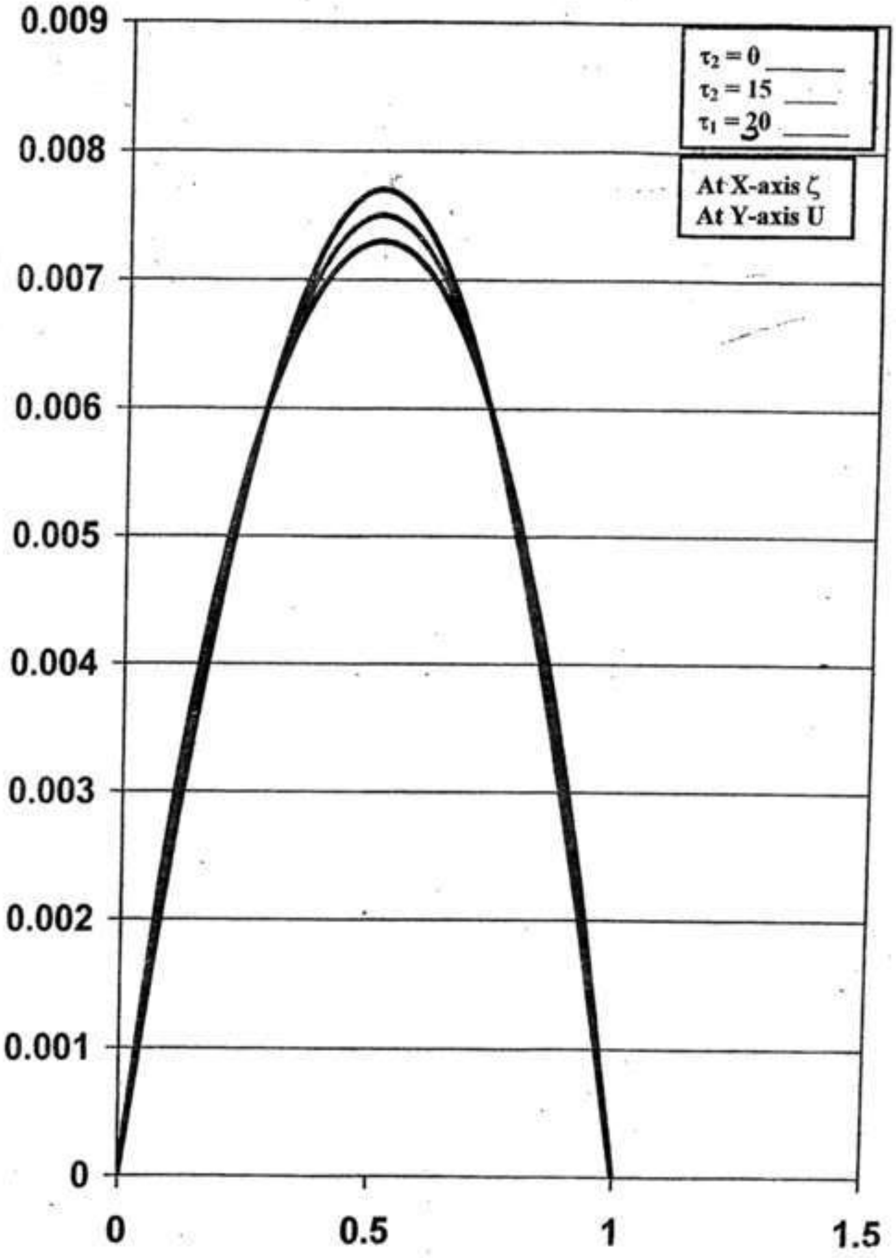


Fig (7) Variation of radial velocity at different cross -viscous parameter at $\tau = \pi/3$.

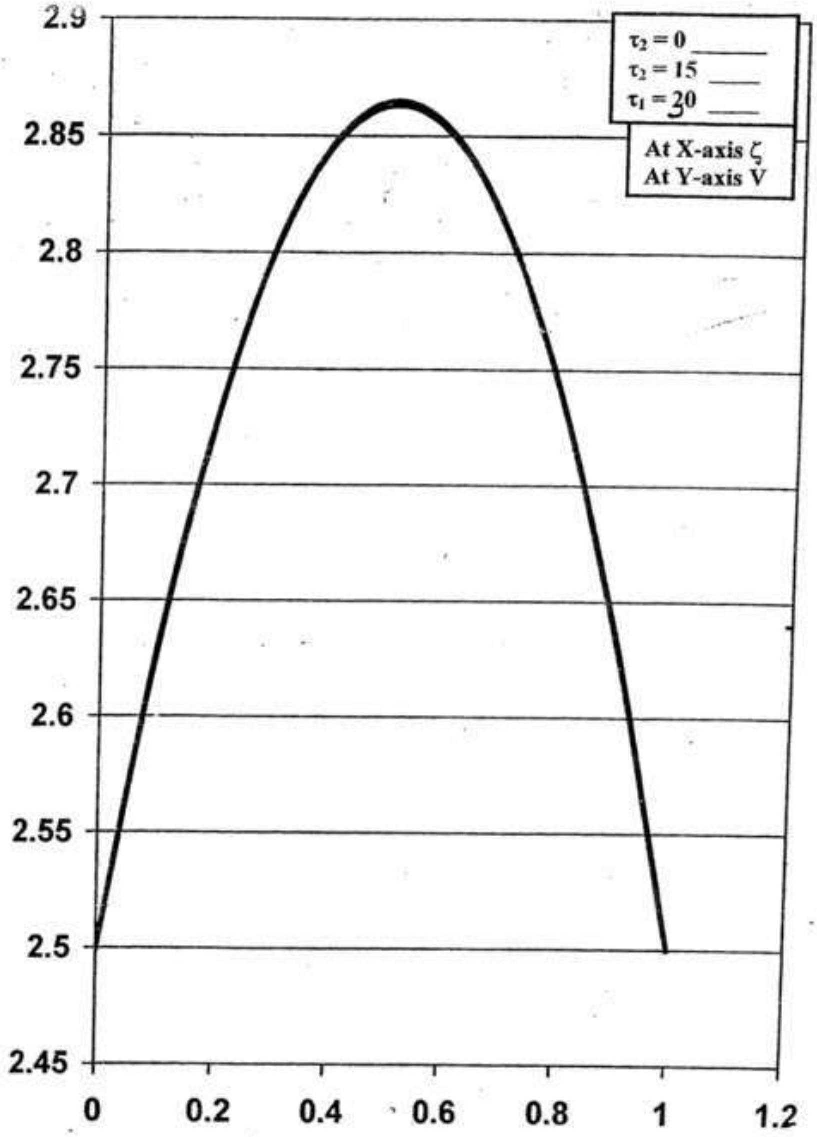


Fig (8) Variation of transverse velocity at different cross -viscous parameter at $\tau = \pi/3$.

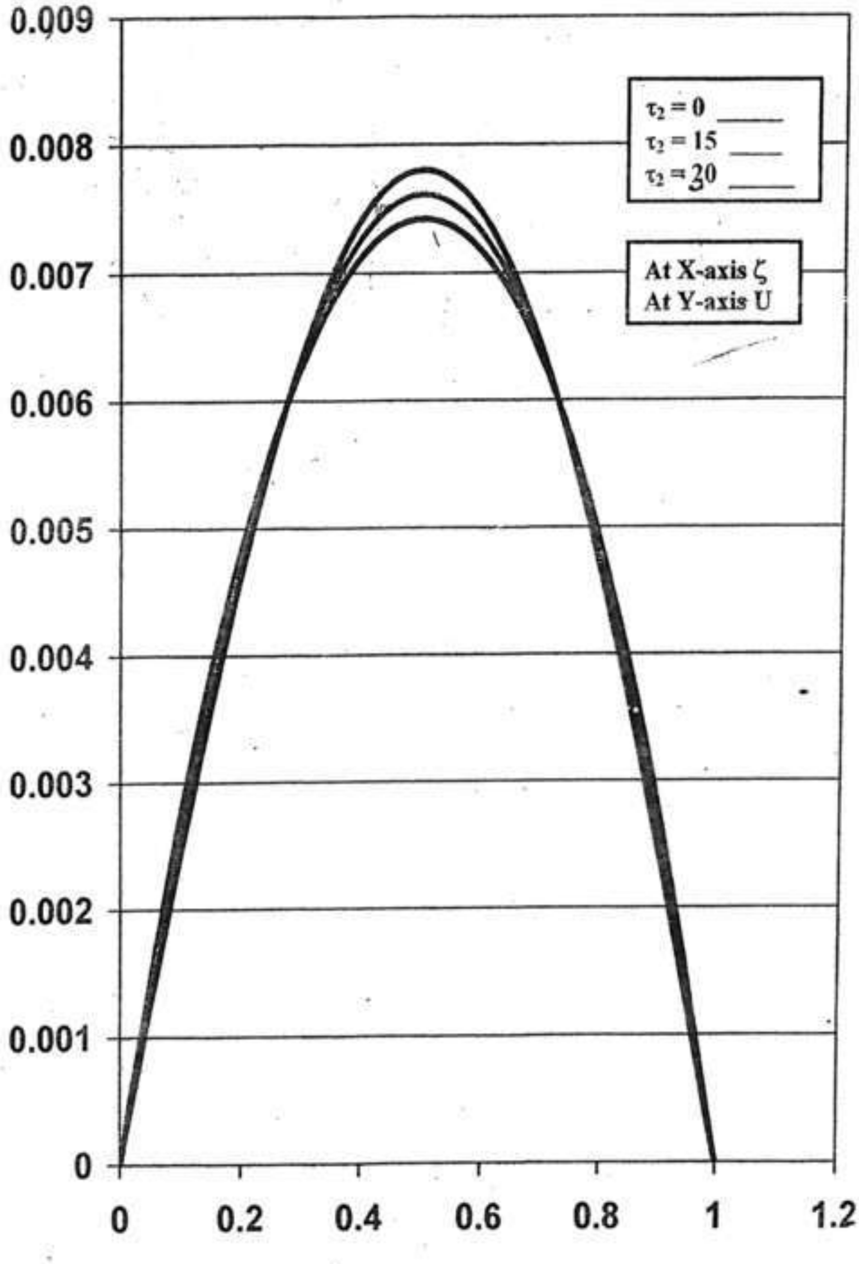


Fig (10) Variation of radial velocity at different cross -viscous parameter at $\tau = 0$.

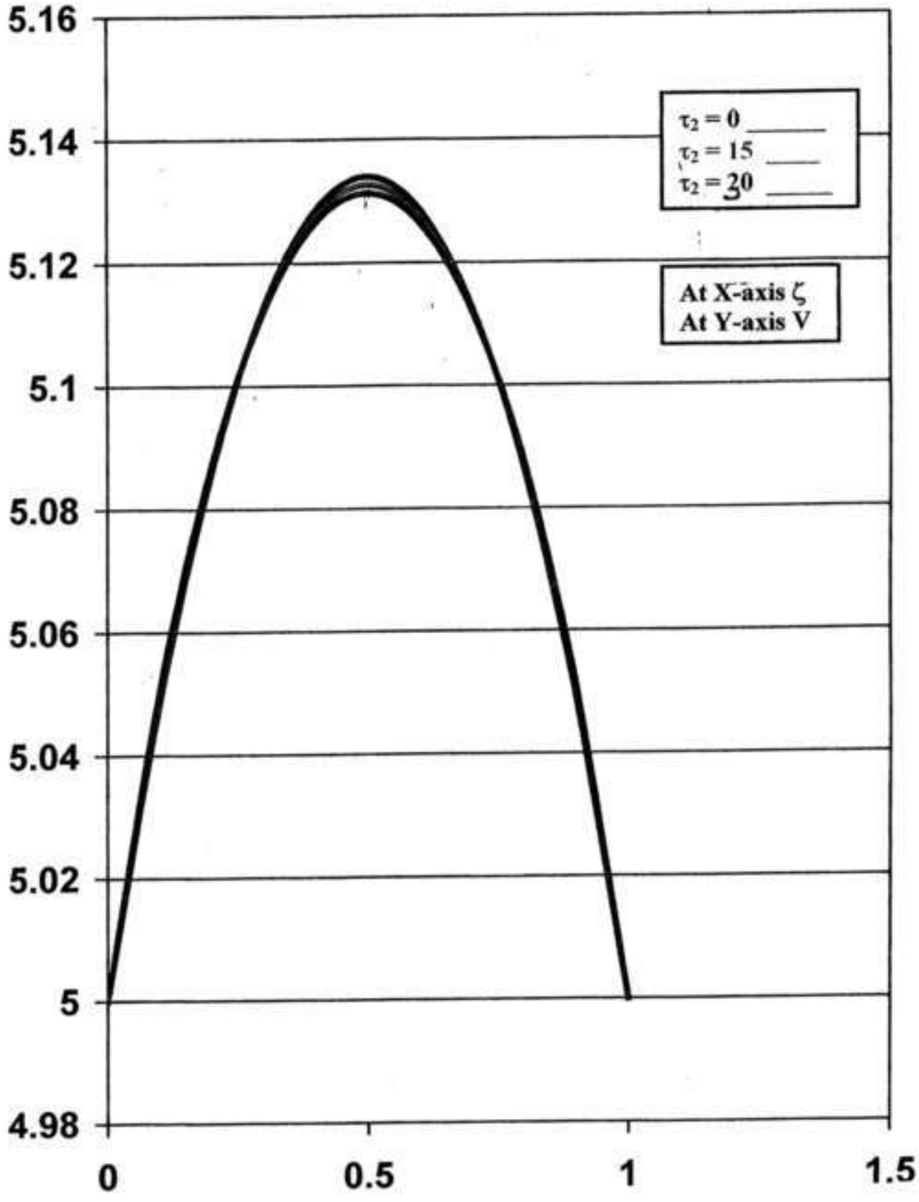


Fig (11) Variation of transverse velocity at different cross -viscous parameter at $\tau = 0$.

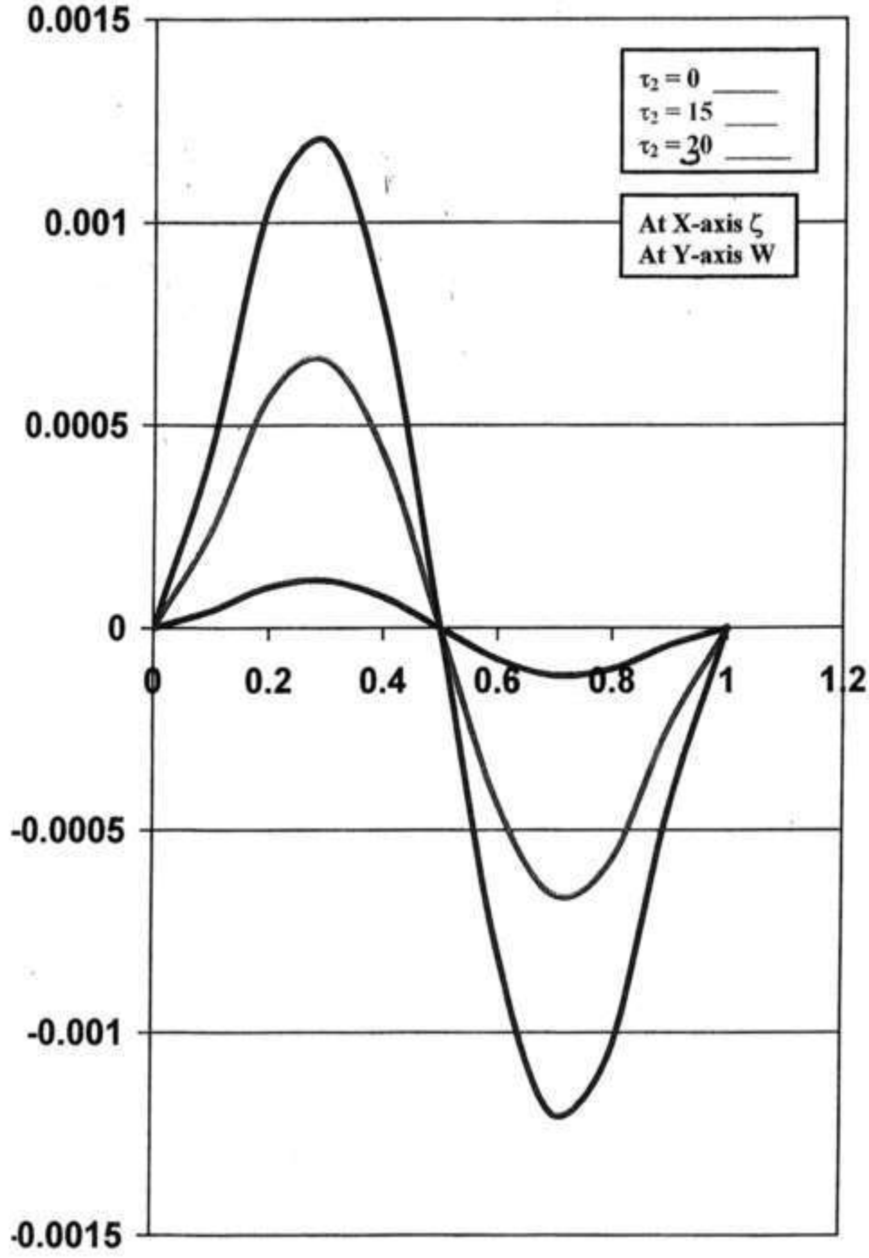


Fig (12) Variation of axial velocity at different cross -viscous parameter at $\tau = 0$.

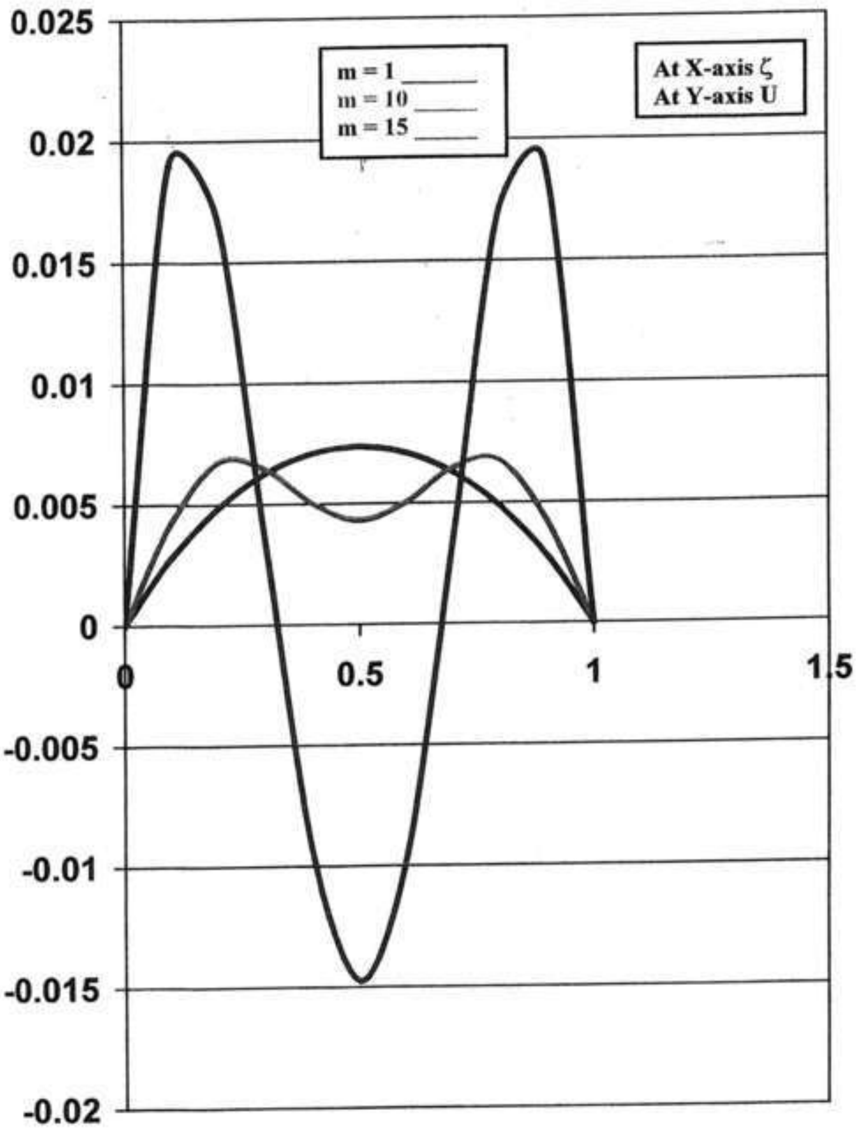


Fig (13) Variation of radial velocity at different magnetic field at $\tau = \pi/3$

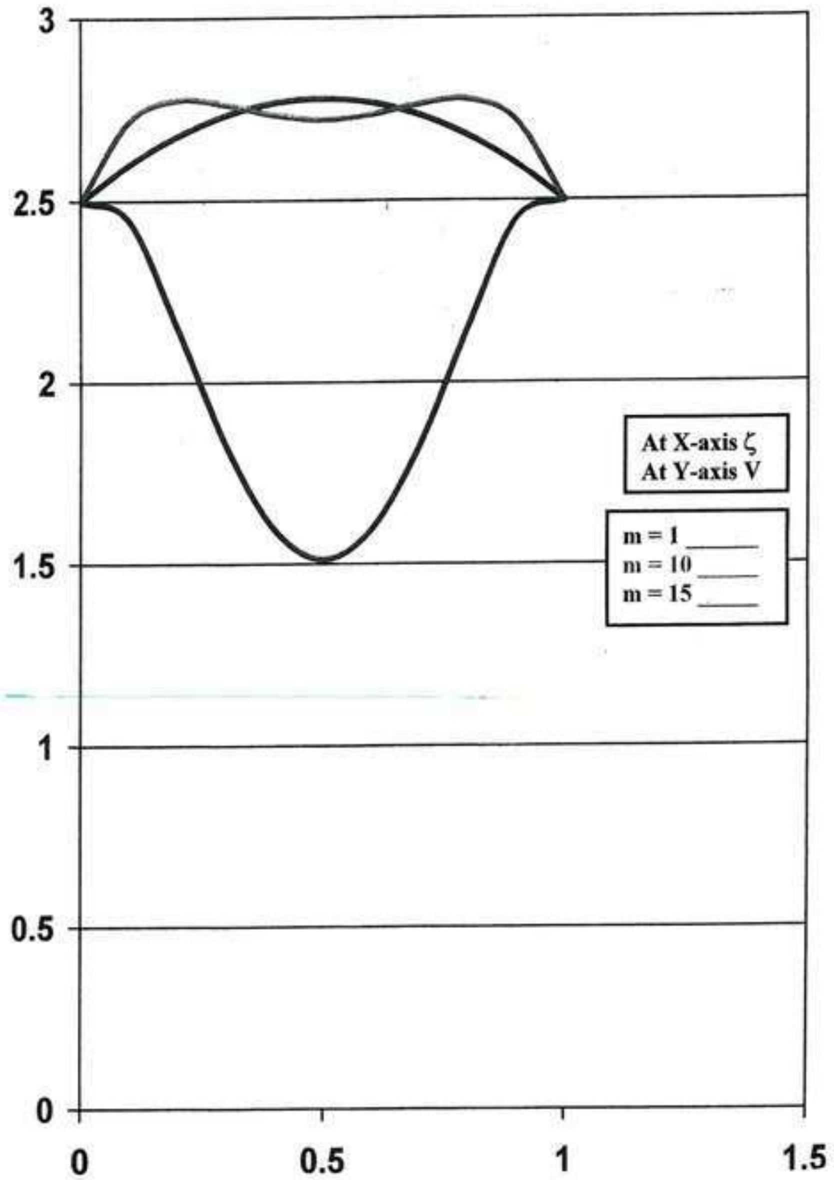


Fig (14) Variation of transverse velocity at different magnetic field at $\tau = \pi/3$

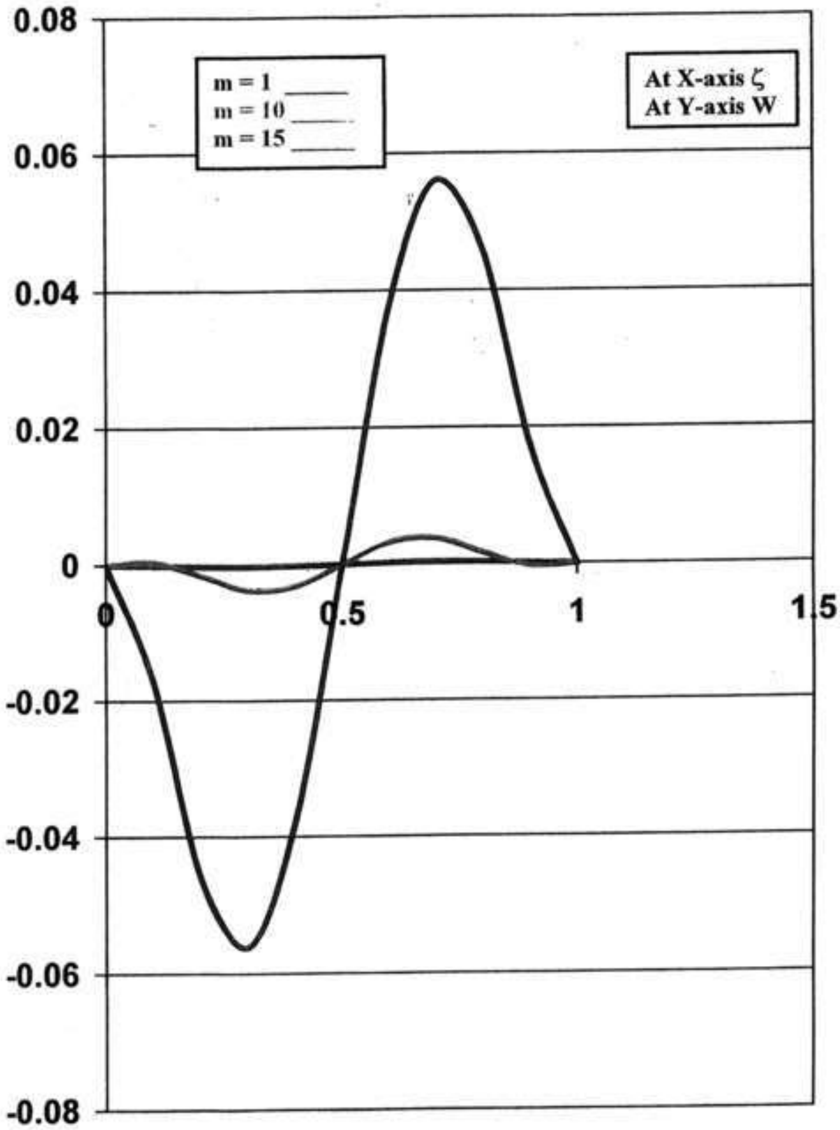


Fig (15) Variation of axial velocity at different magnetic field at $\tau = \pi/3$

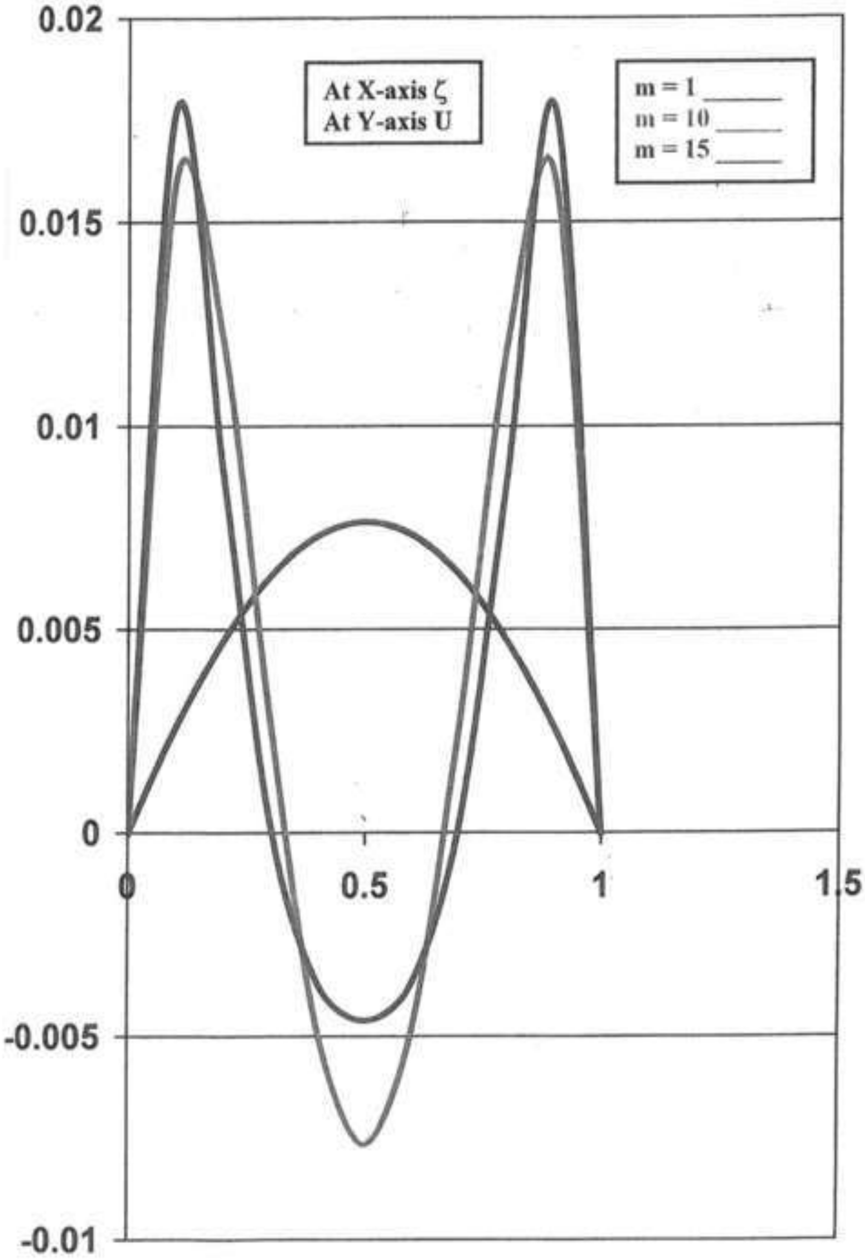


Fig (16) Variation of radial velocity at different magnetic field at $\tau = 0$

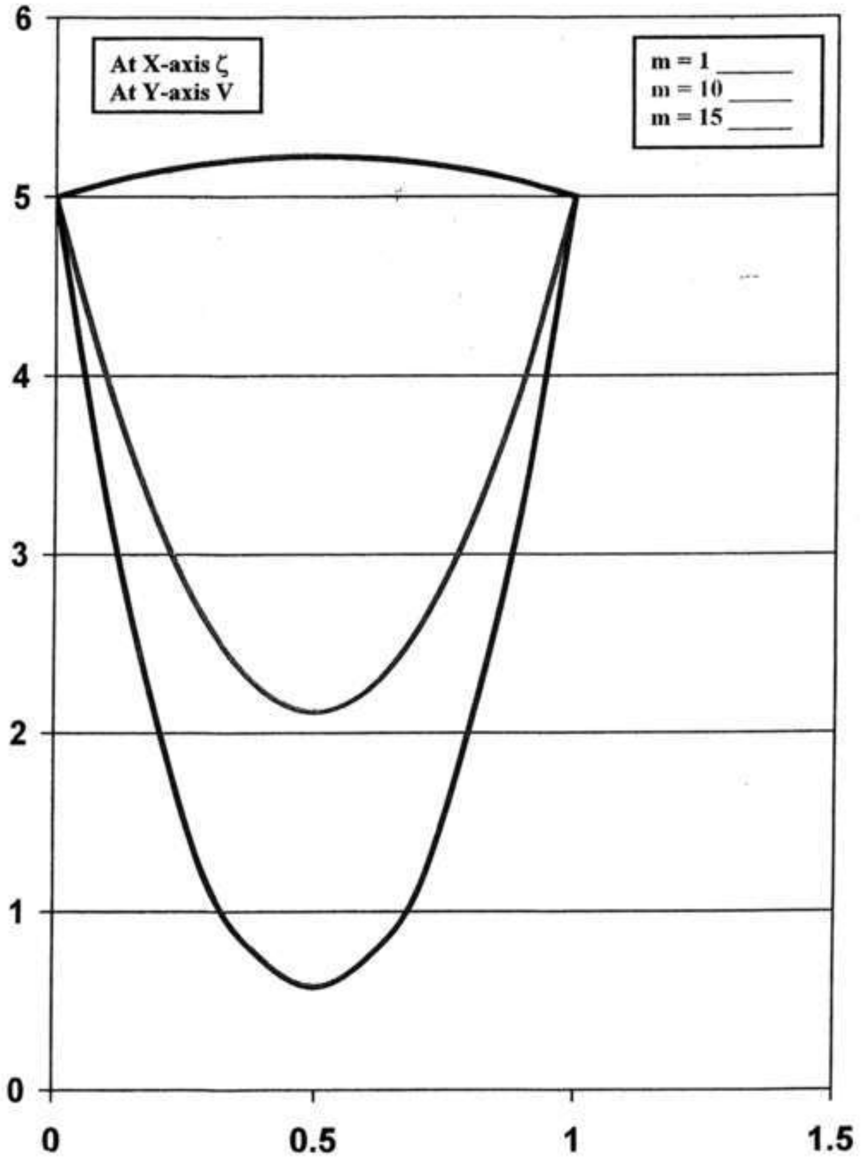


Fig (17) Variation of transverse velocity at different magnetic field at $\tau = 0$

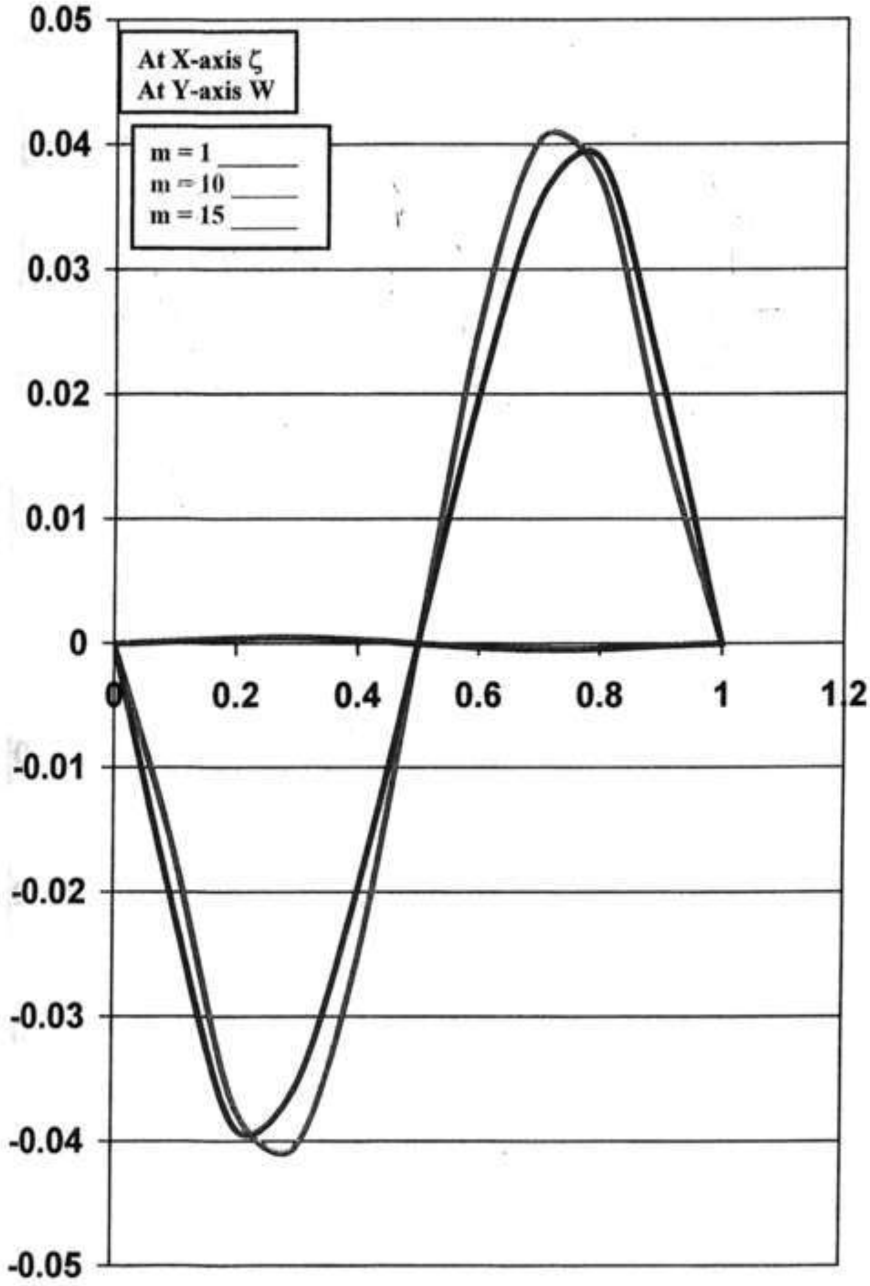


Fig (18) Variation of axial velocity at different magnetic field at τ = 0.

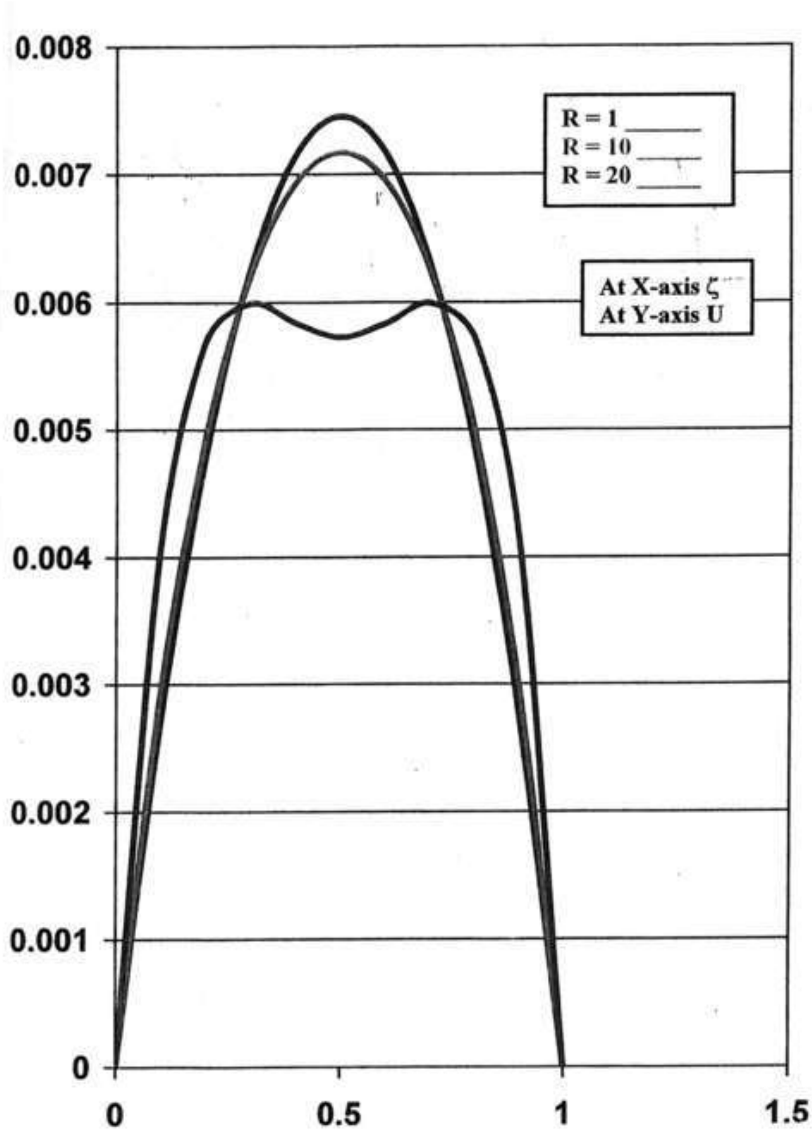


Fig (19) Variation of radial velocity at different Reynolds number at $\tau = \pi/3$

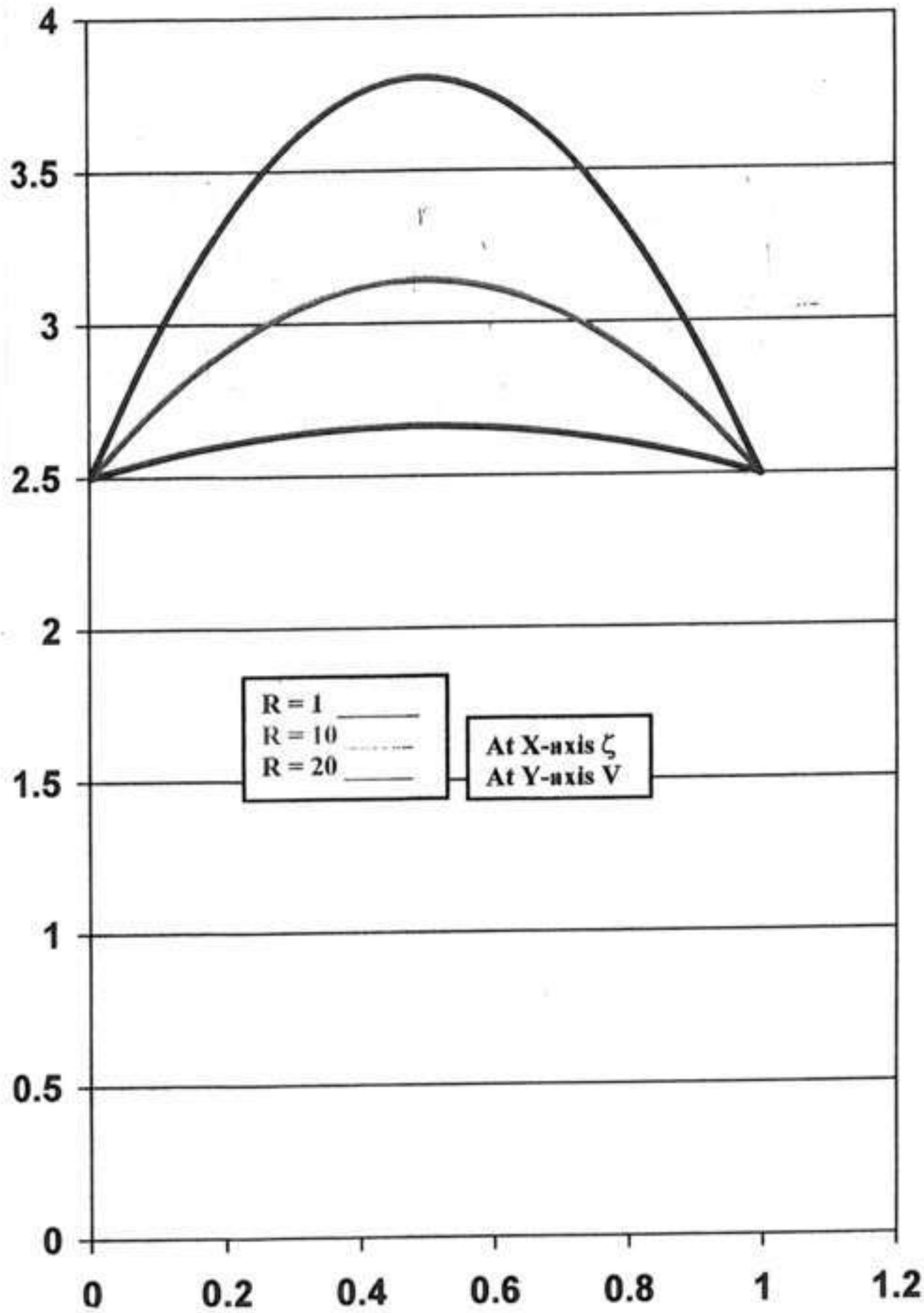


Fig (20) Variation of transverse velocity at different Reynolds number at $\tau = \pi/3$

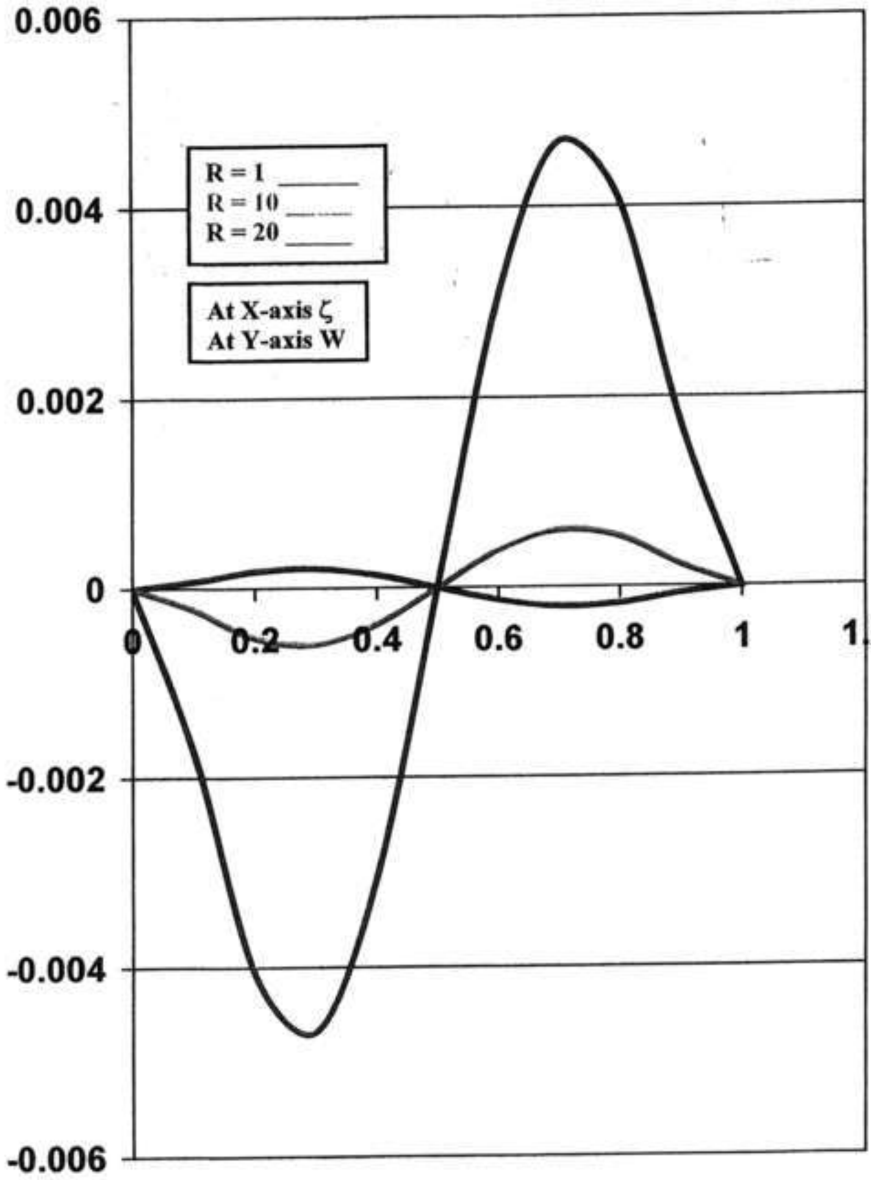


Fig (21) Variation of axial velocity at different Reynolds number at $\tau = \pi/3$

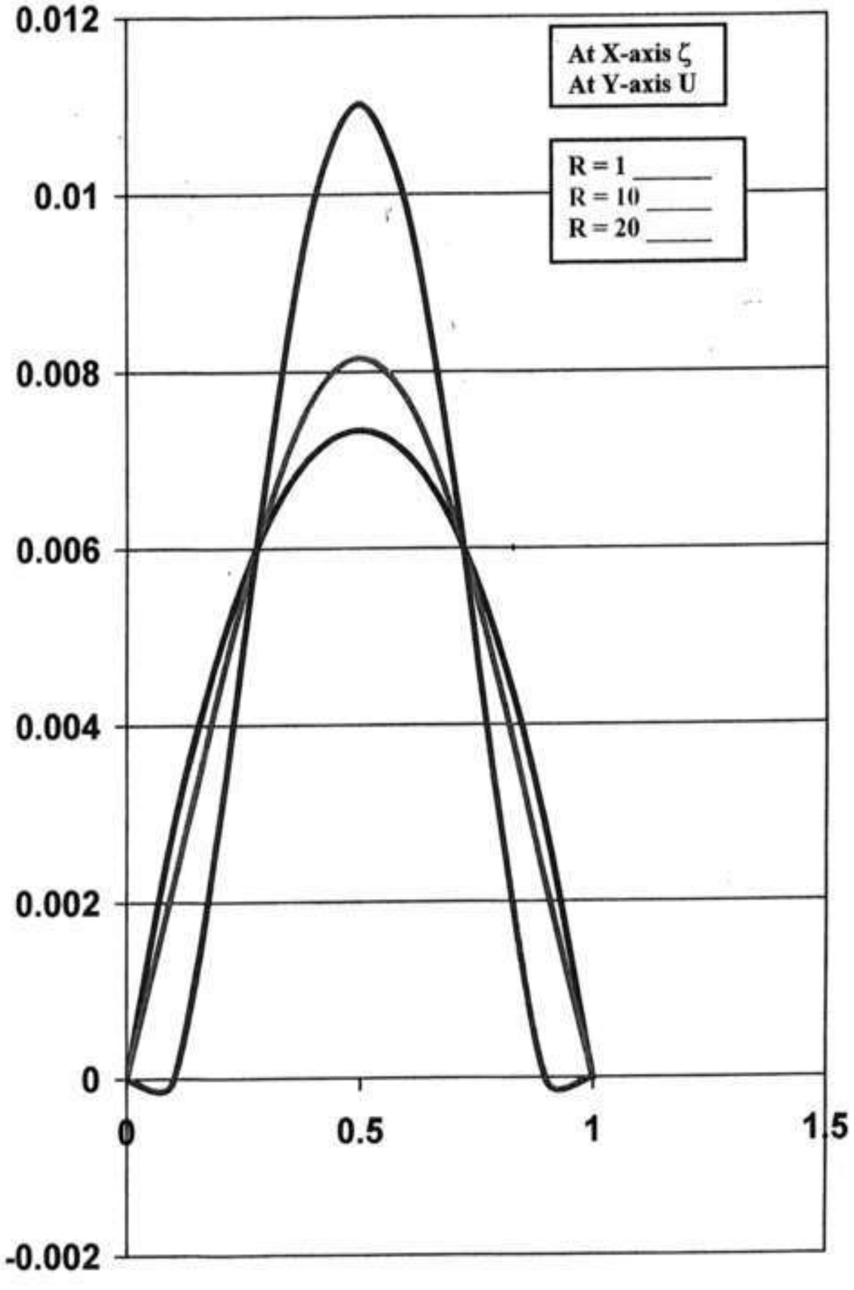


Fig (22) Variation of radial velocity at different Reynolds number at $\tau = 0$

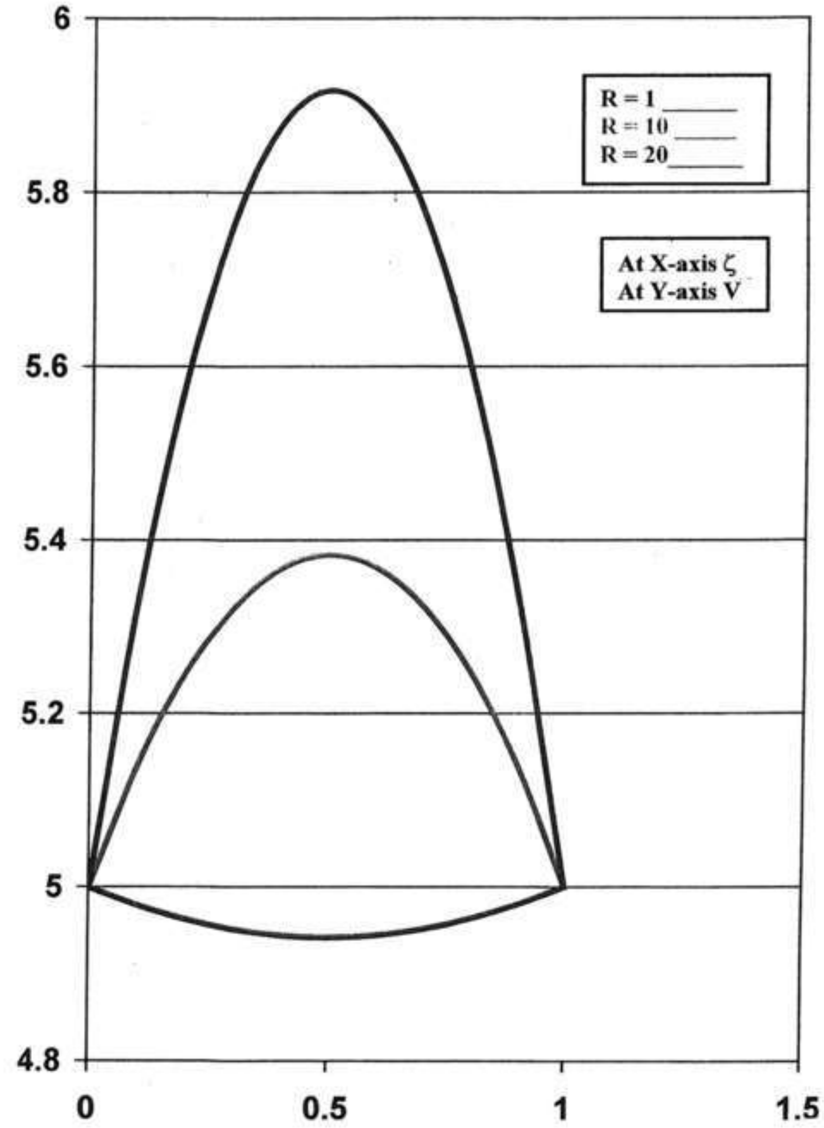


Fig (23) Variation of transverse velocity at different Reynolds number at $\tau = 0$

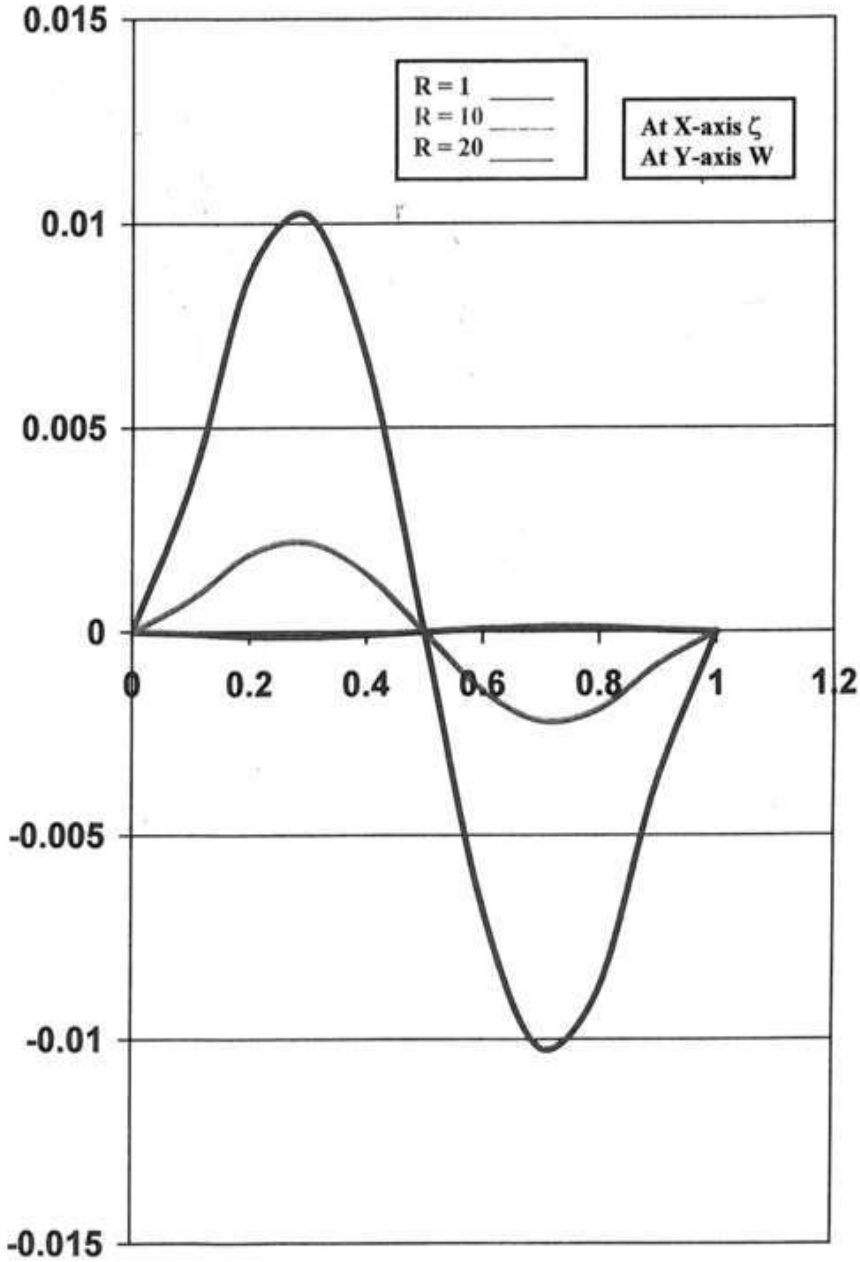


Fig (24) Variation of axial velocity at different Reynolds number at $\tau = 0$

Chapter No. 5

HEAT TRANSFER IN THE FLOW OF A SECOND-ORDER FLUID THROUGH A CHANNEL WITH POROUS WALLS UNDER A TRANSVERSE MAGNETIC FIELD

V.1 INTRODUCTION

The heat transfer in the flow of an electrically conducting fluid between porous boundaries is of practical interest in problems of gaseous diffusion etc. Terrill and Shrestha[73)] have discussed the problem of steady laminar flow of an incompressible viscous fluid in a two dimensional channel when the walls are of different permeability and studied the effects of magnetic field when the fluid is electrically conducting[74)]. The problem of flow of a second-order fluid with heat transfer in a channel with porous walls has been considered by Agrawal[75)]. Sharma & Singh[76)] have studied the numerical solution of the flow of second-order fluid through a channel with porous walls under a transverse magnetic field.

The purpose of the present paper is an attempt to study the heat transfer in the flow of a second-order fluid through a channel with porous walls under a transverse magnetic field by regular perturbation technique. The second-order effects on the temperature profile are illustrated graphically for different values of the Hartman and Reynolds number. The results are also obtained for the Newtonian fluid by taking the second-order parameter to be zero.

V.2 FORMULATION OF THE PROBLEM

The heat transfer in the steady two dimensional flow of an incompressible second-order fluid in a channel, of width 2h consisting of two porous walls (coinciding with the plane $y = \pm h$) of equal permeability is considered. The whole system of the channel is constructed in such a manner that its bottom and top becomes perfectly insulted and does not transmit the heat. A constant magnetic field H_0 is applied normal to the axis of the channel. The induced magnetic field has been neglected in the flow since the magnetic Reynolds number is small. A uniform suction V is applied to the both the walls of the channel. Let us choose with x and y axes respectively in a plane parallel and perpendicular to the channel walls. Let u and v be the components of the velocity in x and y directions respectively.

Following Terrill and Shrestha[73)] a stream function

$$\Psi\ (x,\xi) = (hU-Vx)\, f(\xi)$$

(5.1)

Where U is the entrance velocity and $\xi(= y/h)$ is the dimensionless distance while 2h is the distance between the channel walls. In non-dimensional form the velocity field by Terril and Shrestha[73)] is taken as:

$$U\ (x, \xi) = (U-Vx/h)\, f'\ (\xi)$$

$$v\ (\xi) = V\, F\ (\xi)$$

(5.2)

Where dash denotes differentiation with respect to ξ. The expression (5.2) suggests that u is a function of x and ξ, while v is a function of ξ only. Using this fact, the constitutive equation (1.4) the equation of continuity and momentum equations can be written as:

$$\partial u/\partial x + (1/h)(\ \partial v/\ \partial\xi) = 0$$

(5.3)

$$u\partial u/\ \partial x + (v/h)\ \partial v/\ \partial\xi = -(1/p)\ (\ \partial p/\ \partial x) + (v_1/h^2)(\ (\ \partial^2 u/\ \partial\xi^2) + v_2[(1/h^2 1)$$

$$(\partial^2/\partial\xi^2)\{u\partial u/\partial x+(v/h)(\partial v/\partial\xi)\}+(2/h^2)(\partial/\partial\xi)$$

$$\{(\partial u/\partial x)(\partial v/\partial\xi))\}]+(v_3/h^2)(\partial/\partial x)(\partial u/\partial\xi)^2$$

$$-\mu_e{}^2 H_0{}^2\sigma u/p \qquad (5.4)$$

$$v\partial v/\partial\xi=-(1/p)(\partial p/\partial\xi)+(v_1/h)(\partial^2 v/\partial\xi^2)+v_2[(2/h)(\partial^2/\partial\xi^2)\{(v/h)(\partial v/\partial\xi)\}$$
$$+2(\partial/\partial x)\{(\partial u/\partial x)(\partial u/\partial\xi)\}]+(4/h^2)\{(\partial u/\partial\xi)(\partial^2 u/\partial\xi^2)+(\partial v/\partial\xi)(\partial^2 v/\partial\xi^2)- \quad -$$
$$\partial^2/\partial x\,\partial\xi\{u\partial u/\partial x+(v/h)\,\partial u/\partial\xi\}]+(v_3 h^2)[4\partial/\partial\xi(\partial v/\partial\xi)^2+\partial/\partial\xi(\partial u/\partial\xi)^2]$$
(5.5)

$$pc_v(u\partial T/\partial x+v\partial T/\partial y)=k(\partial^2 T/\partial x^2+\partial^2 T/\partial y^2)+\Phi$$
(5.6)

where p is the density, μ_e is the magnetic permeability, σ is the electric conductivity, v_1 (= μ_1/p) is the kinematic viscosity, v_2 (=μ_2/p) is the kinematic elastic-viscosity, v_3 (= μ_3/p) is the kinematic coefficient of cross-viscosity, c_v is the specific heat at constant volume, k is the thermal conductivity and ξ = y/h is the dimensionless distance.

The viscous dissipation function Φ is given by

$$\Phi = \tilde{\tau}^i{}_j d^i{}_j$$
(5.7)

Where $\tilde{\tau}^i{}_j$ is the mixed deviatoric stress tensor.

The boundary conditions are,

$u(x, \pm 1) = 0$, $\quad (\partial u/\partial\xi)_{\xi=0}=0$,

$v(x,0) = 0$, $\quad v(x,1) = V$, $\quad v(x, -1) =-V$,

$T(x, 1)= T_1$, $\quad T(x,-1) =T_{-1.}$
(5.8)

Substituting (5.2) in equation (5.4) and (5.5) and eliminating p from the obtained equation, we get

$$f^{iv} + R(f' f'' - f f''') + \tau_1 (f f^{v} - f' f^{iv}) - S^2 f''=0, \quad (5.9)$$

where R (= Vh/v_1) is the suction Reynolds number, τ_1 (=v_2V/hv_1) is an elastic-viscous parameter governing the effects of elastic-viscosity of the fluid and S[-$\mu_e H_0 h(\sigma/\mu_1)^{1/2}$] is the Hartmann number.

Equation (5.6) together with equation (5.2) suggests the form of the temperature distribution as0 follows:

$$T = T_{-1}+(v_1V)[\, \phi(\xi)+\{(U/V)-(x/h)\}^2\, \Psi(\xi)]/(hC_v). \quad (5.10)$$

Using equation (5.10) in equation (5.6) and equating the coefficient of $(U/V-x/h)^2$ and terms independent of $(U/V-(x/h)^2$ on both sides of the resulting equation, we obtain

$$\phi''-2RPf\,\phi'+2\Psi + 8RPf'^2+ 8R^2P\tau_2\, f\, f'\, f'' = 0, \quad (5.11)$$

$$\psi''-RPf\,\psi'+4RP\,\psi f' +2RPF''^2+2R^2P\tau_2(ff''f''-f'f''^2)=0. \quad (5.12)$$

Where p = $\mu_1 c_v/k$ is the Prandtl number, $\tau_2=2\mu^2/(h^2p)$ is the second-order parameter.

The expression of the temperature distribution in the dimensionless form can be expressed as:

$$T = (T-T_{-1})/(T_1-T_{-1})=E(\phi+\zeta^2\,\psi), \quad (5.13)$$

where ζ[=(U/V-x/h)] is the dimensionless distance and E(=v_1V/{(T_1-T_{-1})hC_v}] is the Eckert number.

V.3 SOLUTION OF THE PROBLEM

Assuming the relationships τ_1=-R τ_1(τ_1≥0) and $S^2 = RS_1^2$ eqn. (5.9) becomes

$$F^{iv}+R\,(f'\,f''-f\,f''')-R\,\tau_1(f\,f^{v}-f'\,f^{iv})-RS_1^2f\,''=0$$
(5.14)

For small values of the suction Reynolds number R, we can develop a regular perturbation scheme fo solving eqns. (5.11), (5.12) & (5.14) by expanding f, - and – in powers of R. Substituting

$$f(\xi)=\sum R^{n}f_n(\xi)$$
(5.15)

$$\phi(\xi)=\sum R^{n}\phi_n(\xi)$$
(5.16)

$$\psi(\xi)=\sum R^{n}\psi_n(\xi)$$
(5.17)

eqns. (5.11), (5.12) & (5.14)and equating the like powers of R on the two sides of the resulting equations, we obtain the following sets of equations:

$$f_0^{iv}=0$$

$$f_1^{iv}+f_0'\,f_0''-f_0f_0'''-\tau_1\,(f_0\,f_0^{v}-f_0'f_0^{iv})-S_1^2f_0''=0$$

$$f_2^{iv}+f_1'\,f_0''-f_0'f_1''-f_1'f_0'''-\tau_1\,(f_1\,f_0^{v}-f_0'f_1^{v}-f_1'f_0^{iv}-f_0'f_1^{iv})-S_1^2f_1''=0$$
(5.18)

$$\psi_0=0$$

$$\psi_1=2Pf_0\,\psi_0'+4P\psi_0f_0'+2Pf_0''^{2}=0$$

ψ_2= 2P($f_1\ \psi_0$'+$f_0\ \psi_1$')+4P($\psi_0 f_1$'+f_1'ψ_0+f_0''$f_/$'')+2P τ_2(f_0-f_0''f_0'''-f_0'f_0''^{2})=0

(5.19)

ϕ_0''+2ψ_0 =0,

ϕ_1''-2P$f_0\ \phi_0$'+2ψ_1+8Pf_0'^{2}=0,

ϕ_2''-2P($f_1\ \phi_0$'+ $f_1\ \phi_0$')+2 ψ_{2v}+16P $f_0\ 'f_1$'+8 $\tau_2\ f_0\ f_0$' f_0'''=0

(5.20)

Boundary condition (5.8) can be rewritten as:

$f_n(0) = f_n'(1) = f_n''(0) = 0\ \forall n$

$f_0(1) = 1$, $\mathbf{f_n}(1) = 0 \geq 1$

$\phi_n(-1) = 0\ \forall n$, $\phi_0(1) = 1/E = w$(say),

$\phi_n(1) = 0$, $0 \geq 1$, $\psi_n(\pm 1) = 0\ \forall n$

The solution of equation (5.18), (5.19), (5.20) subjected to the boundary condition (5.21) is given as follows:

$f_0(\xi) = (1/2)(3\xi - \xi^3)$,

$f_1(\xi)$ = -(1/280)(ξ^7-3 ξ^3+2 ξ)-S_1^2/40)(ξ^5-2 ξ^3+ ξ),

$f_2(\xi)$ = (1/1293600)(14 ξ^{11}-385 ξ^9+198 ξ^7+876 ξ^3-703 ξ)-(τ_1/280) {(3ξ^7-9ξ^3+6ξ)+S_1^2(ξ^7-3ξ^3+2ξ)}-S_1^2({1/100800)(15ξ^9+108ξ^7-54ξ^5-276ξ^3+207ξ) +(S_1^2/8400)(5ξ^7-21ξ^5+27ξ^3-11ξ)}.

$\psi_0(\xi) = 0$,

$\psi_1(\xi) = (3/2)P(1-\xi^4)$,

$\psi_2(\xi)$ = $3P^2\{383/280-\xi^8/56-\xi^6/10+\xi^4/4-(3/2)\xi^2)$ $-P\{(9/280)$ $(1-\xi^4)^2$ $+(S_1^2/10)(1+2\xi^6-3\xi^4)\}-(3/5)P\tau_2(1-\xi^6)$

$\phi_0(\xi) = (w/2)(\xi+1),$

$\phi_1(\xi) = (wP/40)(10\xi^3 - \xi^5-9\xi) - (P/2)(21\xi^2+\xi^6-6\xi^4-16)$

$\phi_2(\xi)$=$P^2[29\xi^{10}/840-51\xi^8/140+37\xi^6/20-9\xi^4/2-1149\xi^2/280+(w/40)$ $(1391\xi/2520-9\xi^3/2+99\xi^5/20-15\xi^7/14+5\xi^9/72)]-P[11/168-33\xi^2/280+11\xi^4$ $/140-3\xi^6/140-3\xi^8/280+\xi^{10}/168-S_1^2(2\xi^2/5-13\xi^8/280+\xi^6/5-7\xi^4/20-57/280)$ $+\tau_2(3-3\xi^2/5-3\xi^8/10+12\xi^6/5-9\xi^4/2)-w\{(71\xi/100800-\xi^3/840+3\xi^5/5600-$ $\xi^9/20160)+S_1^2(19\xi/8400-\xi^7/1680+\xi^5/400-\xi^3/240)\}].$

V.4 RESULTS AND DISCUSSIONS

(i) The values of the functions f_0, f_1 and f_2 are identical to those obtained by Sharma and Singh[76)].

(ii) For $\tau_2 = 0$ the results are in good agreement with those obtained by Terril and Shrestha[73)].

(iii) For S = 0 the results are matching with those obtained by Agarwal[75)].

The variation of the temperature profile at P = 0.4, $\zeta = 0.4$, E = 1, $S_1 = 1$, $\tau_2 = -1$ for R = 0.01, 0.1, 1.0 is represented in fig (1). It is evident that for R = 0.1, temperature increases with ξ upto $\xi = 0.7$ approximately and thereafter decreases very slowly and attains its value 1 at the boundary wall $\xi = 1$. At the R = 1 the temperature graph is parabolic with vertex upward and attains its maximum value at the middle of the wall gap-length with minimum at the boundary wall $\xi = -1$. At R = 0.01, Temperature increases linearly throughout the wall gap-length with minimum at the boundary wall $\xi = -1$ and maximum at $\xi = 1$ It is also clear from this figure that the temperature increases with an increase in suction Reynolds number R.

The variation of the temperature profile at P = 0.4, ζ = 0.4, E = 1, S_1 = 1, R = 1, for τ_2 = 0, 0.1, 1.0 is represented in fig (2). It is evident from this figure that temperature graph is approximately parabolic with vertex upward and attains its maximum value at the middle of the wall gap-length with minimum at the boundary wall ξ = -1. It is also observed from this figure that the temperature decreases with an increase in cross-viscous second-order parameter τ_2.

The variation of the temperature profile at P = 0.4, ζ = 0.4, E = 1, R = 1, τ_2 = -1 for S_1= 0, 1, 2 is represented in fig (3). It is seen from this figure that the temperature graph is approximately parabolic with vertex upward and attains its maximum value at the middle of the wall gap-length with minimum at the boundary wall ξ = -1. It is also observed from this figure that the temperature decreases with an increase Hartman number S_1.

ξ	R = 0.01	R = 0.1	R = 1.0
1	1	1	1
0.9	0.955038	1.005515	2.019143
0.8	0.909943	1.009024	2.949132
0.7	0.864639	1.009624	3.769926
0.6	0.819024	1.006305	4.469361
0.5	0.772992	0.998043	5.042745
0.4	0.726434	0.983871	5.491384
0.3	0.679251	0.962928	5.820534
0.2	0.631358	0.934501	6.037138
0.1	0.582687	0.898046	6.147726
0	0.533196	0.853207	6.156709
-0.1	0.482865	0.799824	6.065293
-0.2	0.431702	0.737937	5.871103
-0.3	0.379738	0.667787	5.568581
-0.4	0.327028	0.589804	5.150090
-0.5	0.273648	0.504598	4.607647

-0.6	0.219688	0.412934	3.935044
-0.7	0.165246	0.315694	3.130129
-0.8	0.110425	0.213835	2.196891
-0.9	0.055318	0.108314	1.146950
-1.0	0	0	0

Table (1) Variation of the temperature T* With ξ for different values of Reynolds Number (R).

ξ	$\tau_2 = 0.0$	$\tau_2 = 0.1$	$\tau_2 = 1.0$
1	1	1	1
0.9	1.718003	1.687889	1.416863
0.8	2.380152	2.323254	1.811173
0.7	2.979798	2.900785	2.189671
0.6	3.509659	3.413688	2.549956
0.5	3.962914	3.854930	2.883082
0.4	4.333768	4.218007	3.176152
0.3	4.617650	4.497361	3.414766
0.2	4.81116	4.688561	3.585181
0.1	4.911904	4.788322	3.676084
0	4.918309	4.794469	3.679909
-0.1	4.829472	4.705890	3.593651
-0.2	4.645125	4.522527	3.419146
-0.3	4.365696	4.245408	3.162812
-0.4	3.992474	3.876713	2.834858
-0.5	3.527816	3.419832	2.447984
-0.6	2.975341	2.879370	2.015638
-0.7	2.340001	2.260988	1.549873

-0.8	1.627911	1.571013	1.058932
-0.9	0.845811	0.815697	0.544671
-1.0	0	0	0
Table (2) Variation of the temperature T* with ξ for different values of τ_2			

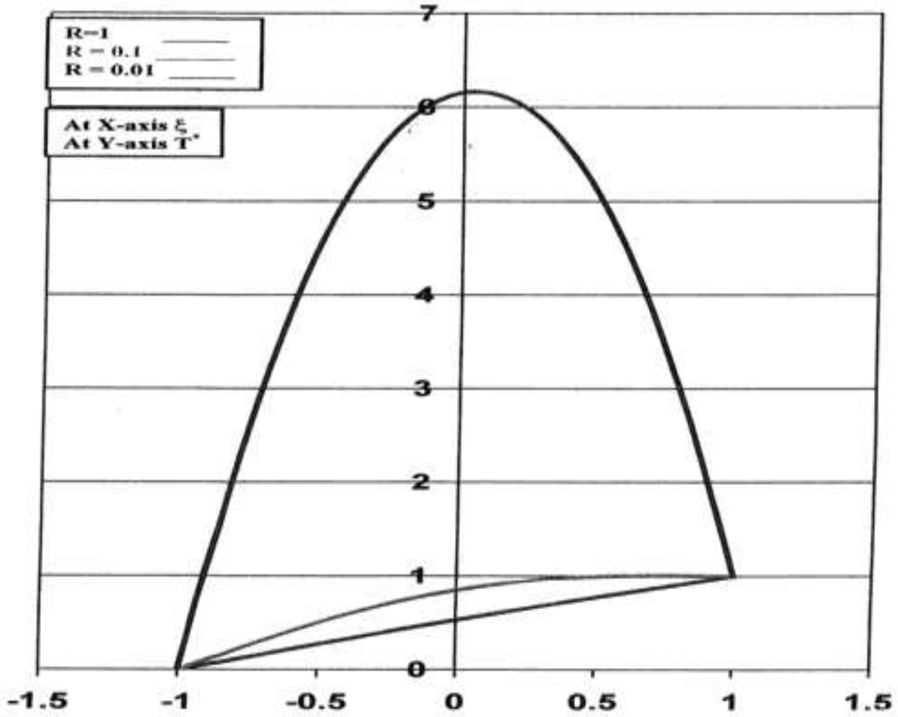
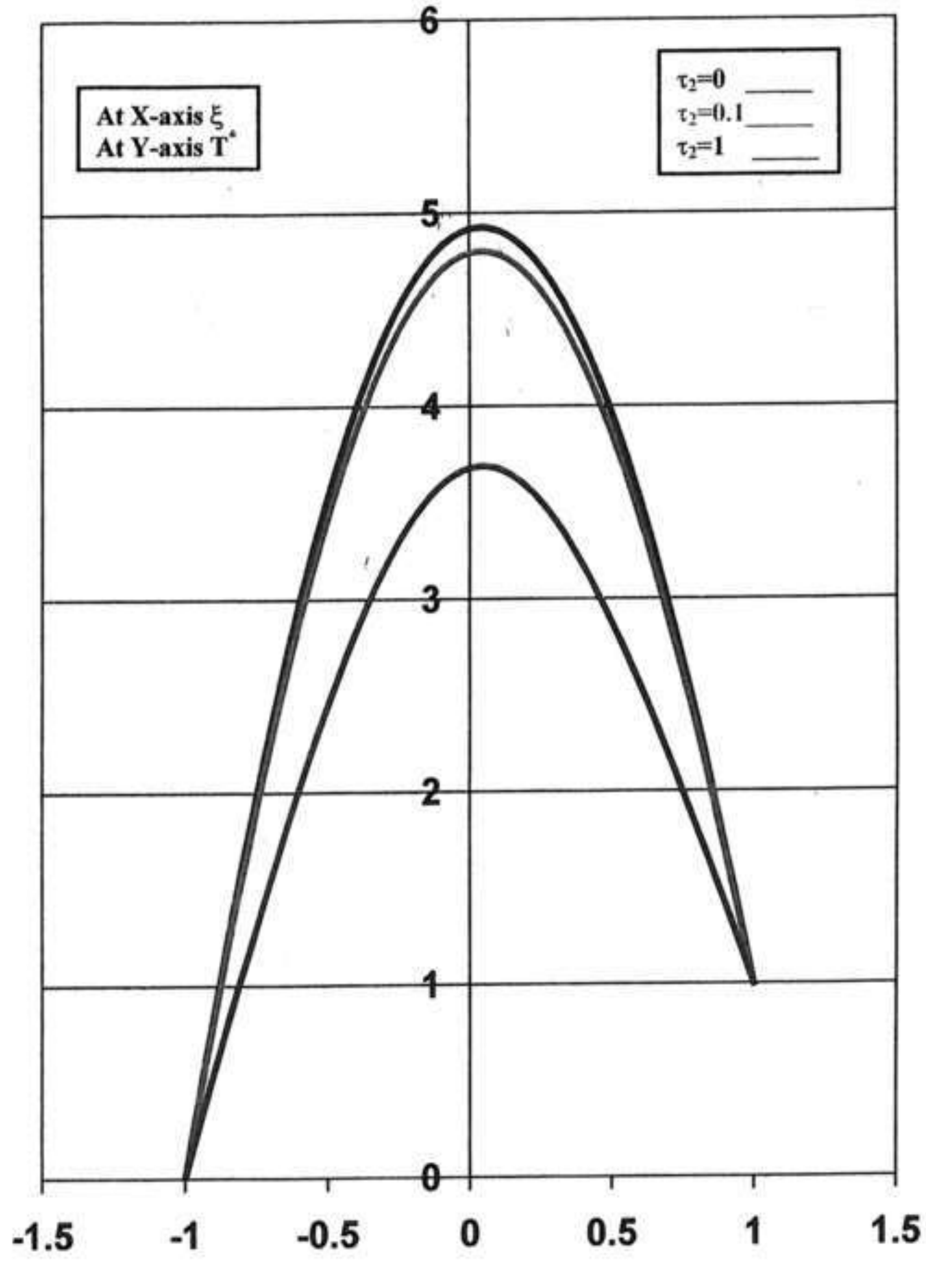


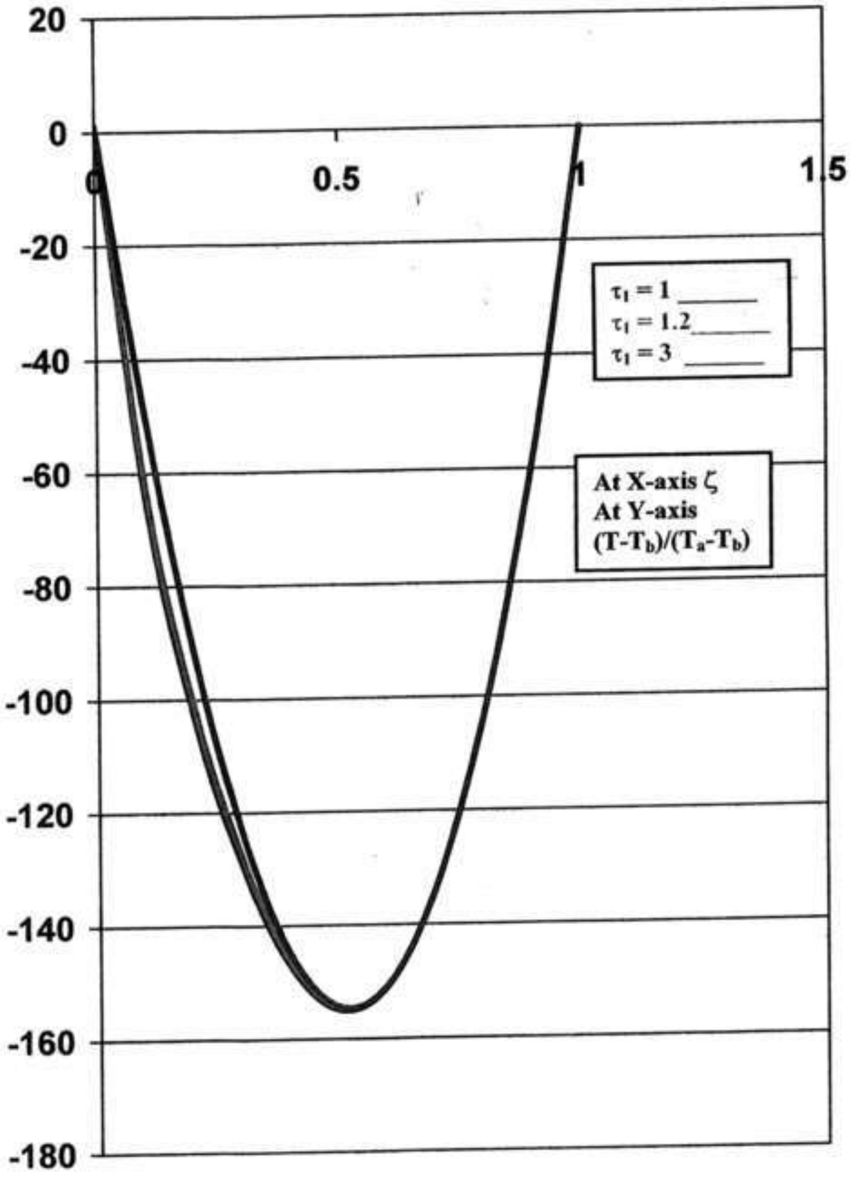
Fig (1) Variation of the temperature T^* With ξ for different values of Reynolds Number R.

ξ	S1 = 0.0	S1 = 1.0	S1 = 2.0
1	1	1	1
0.9	2.028834	2.019143	1.990070
0.8	2.969391	2.949132	2.888353
0.7	3.801313	3.769926	3.675765
0.6	4.512168	4.469361	4.340941
0.5	5.096872	5.042745	4.880362
0.4	5.556177	5.491384	5.297007
0.3	5.894664	5.820534	5.598143
0.2	6.118587	6.037138	5.792790
0.1	6.233877	6.147726	5.889271
0	6.244537	6.156709	5.893224
-0.1	6.151622	6.065293	5.806305
-0.2	5.952888	5.878581	5.625747
-0.3	5.643168	5.568581	5.344817
-0.4	5.215413	5.150090	4.954122
-0.5	4.662322	4.607647	4.443624
-0.6	3.978358	3.935044	3.805099
-0.7	3.161936	3.130129	3.034707
-0.8	2.217447	2.196891	2.135222
-0.9	1.156793	1.146950	1.117422
-1.0	0	0	0

Table (3) Variation of the temperature T* with ξ for different values of Hartman Number (S_1)



Fig(2) Variation of the temperature T^* with ξ for different values of τ_2.



Fig(1) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different elastico-viscous parameter τ_1 at $\tau = \pi/3$.

Chapter No. 6

HEAT TRANSFER IN THE FLOW OF A NON-NEWTONIAN SECOND-ORDER FLUID OVER AN ENCLOSED TORSIONALLY OSCILLATING DISCS WITH UNIFORM SUCTION AND INJECTION IN THE PRESENCE OF THE MAGNETIC FIELD

VI.1 INTRODUCTION

The phenomenon of flow of the fluid over an enclosed torsionally oscillating disc (enclosed in a cylindrical casing) has important engineering applications. The most common practical application of it is the domestic washing machine and blower of curd etc., Soo[64)] has considered first the problem of laminar flow over an enclosed rotating disc in case of Newtonian fluid. Sharma and Agarwal[77)] have discussed the heat transfer from an enclosed rotating disc in case of Newtonian fluid. Thereafter Singh K. R. and H.G. Sharma[78)] have discussed the heat transfer Singh K. R. and H.G. Sharma [78)] have discussed the heat transfer from an enclosed rotating disc in case of Newtonian fluid. Thereafter in the flow of a second-order fluid between two enclosed rotating discs. The torsional oscillations of Newtonian fluids have been discussed by Rosenblat[65)]. He has also discussed the case when the Newtonian fluid is confined between two infinite torsionally oscillating discs[66).] Sharma & Gupta[67)] have considered a general case of flow of a second-order fluid between two infinite torsionally oscillating discs. Thereafter Sharma & K. R. Singh[79)] have solved the problem of heat transfer in the flow of non-Newtonian second-order fluid between torsionally oscillating plane Riley & Wybrow[71)] have considered the flow induced by the torsional oscillations of an elliptic cylinder. Sadhna kahre[61)] studied the steady flow between a rotating and porous stationary disc in the presence of transverse magnetic field.

Due to complexity of the differential equations and tedious calculations of the solutions, no one has tried to solve the most practical problems of enclosed torsionallly oscillating discs so far. The authors have considered the present problem of heat transfer in the flow of a non-Newtonian second-order fluid over an enclosed torsionally oscillating discs with uniform suction and injection in the presence of the magnetic field and calculated successfully the steady and unsteady part both of the flow and energy functions. The flow and energy functions are expanded in the powers of the amplitude $\in$ (assumed to be small) of the oscillations of the disc. The non-Newtonian effects are exhibited through two dimensionless parameters τ_1 ($=n\mu_2/\mu_1$) and τ_2($=n\mu_3/\mu_1$), where μ_1, μ_2, μ_3 are coefficient of Newtonian viscosity, elastic-viscosity and cross-viscosity respectively, n being the uniform frequency of the oscillation. The variation of temperature distribution with elastic-viscous parameter τ_1, cross –viscous parameter τ_2 (based on the relation $\tau_1 = a\,\tau_2$, where a = -0.2 as for 5.46% poly-iso- butylenes type solution in cetane at 30^{0}C (Markowiz[38]) Reynolds number R_1 magnetic field m, suction parameter k at different phase difference τ is shown graphically.

VI.2 FORMULATION OF THE PROBLEM

In the three dimensional cylindrical set of co-ordinates (r, θ, z) the system consists of a finite oscillating disc of radius r_s (coinciding with the plane z = 0) performing rotator oscillations of the type rΩCost of small amplitude ϵ , about perpendicular axis r = 0 with a constant angular velocity Ω in an incompressible second-order fluid forming the part of a cylindrical casing or housing. The top of the casing (coinciding with the plane $z = z_0 < r_s$) may be considered as a stationary disc (stator) placed parallel to and at a distance equal to gap length z_0 from the oscillating disc. The symmetrical radial steady outflow has a small mass rate 'm' of radial outflow ('**-m**' for net radial inflow). The inlet condition is taken as a simple radial source flow along z-axis starting from radius r_0. A constant magnetic field B_0 is applied normal to the plane of the oscillating disc. The induced magnetic field is neglected. The lower disc $z = 0$ is maintained at constant temperature T_a while the upper disc $z = z_a$ at constant temperature T_b.

Assuming (u, v, w) as the velocity components along the cylindrical system of axes (r, θ, z) the relevant boundary conditions of the problem are:

$z = 0$, $u = 0$, $v = r\Omega^{i\tau}$(Real part), $w = w_0$ $T = T_a$

$z = z_0$, $u = 0$, $v = 0$, $w = w_0$ $T = T_b$

(6.1)

where the gap z_0 is assumed small in comparison with the disc radius r_s. The velocity components for the axisymmetric flow compatible with the continuity criterion can be taken as [64,65,66],

$U = - \xi H'(\zeta,\tau)+(R_m/R_z)\, M'(\zeta,\tau)/\xi,$

$V = \xi G'(\zeta,\tau)+(R_l/R_z)\, L(\zeta,\tau)/\xi,$

$W = 2H(\zeta,\tau).$

(6.2)

and for the temperature, we take

$T = T_b+(v_1\Omega/C_v)\{\phi(\zeta,\tau)+\xi^2\Psi(\zeta,\tau)\}$

(6.3)

where $U = u/\Omega z_0, W = w/\Omega z_0$, $\xi = r/\Omega z_0$, ζ,τ are dimensionless quantities and $H(\zeta,\tau)$, $G(\zeta,\tau)$, $L(\zeta,\tau)$, $M'(\zeta,\tau)$, $\phi(\zeta,\tau)$, $\Psi(\zeta,\tau)$ are dimensionless function of the dimensionless variables $\zeta = z/z_0$ and τ = nt. $R_m(=m/2\pi p z_0 v_1)$, R_L $(=\mathbf{L}/2\pi\ p z_0 v_1)$ are dimensionless number to be called the Reynolds number of net outflow and circulatory flow respectively. R_z ($=\Omega z_0^2/\nu_1$) be the flow Reynolds number. The small mass rate 'm' of the radial outflow is represented by

$$\mathbf{m} = 2\pi\rho \int_0^{z_0} r\, u\, dz \qquad (6.4)$$

Using expression (6.2) and (6.3), the boundary conditions (6.1) transform for G, L & H into the following form:

$$G(0,\tau) = \mathrm{Real}(e^{i\tau}),\quad G(1,\tau) = 0,$$

$$L(0,\tau) = 0, \qquad L(1,\tau) = 0,$$

$$H(0,\tau) = k, \qquad H(1,\tau) = 0,$$

$$H'(0,\tau) = 0, \qquad H'(1,\tau) = 0,$$

$$\phi(0,\tau) = 1/E = S, \quad \phi(1,\tau) = 0,$$

$$\Psi(0,\tau) = 0, \qquad \Psi(1,\tau) = 0 \qquad (6.5)$$

where E [$=\Omega\nu_1/\{C_v(T_a-T_b)\}$] is the Eckert number and k [$=w_0/2\Omega z_0$] is the suction parameter.

The conditions on M on the boundaries are obtainable form the expression (6.4) for m as follows:

$M(1, \tau) - M(0, \tau) = 1$

(6.6)

which on choosing the discs as streamlines reduces to

$M(1, \tau) = 1, \qquad M(0, \tau) = 0$

(6.7)

Using eqs. (1.4) and expression (6.2) in equation (1.8) and neglecting the squares

& higher powers of R_m/R_z (assumed small), we have the following equations in dimensionless form:

$$-(1/pz_0)(\partial p/\partial \xi)=n\Omega z_0\{\xi\partial H'-(R_m/R_z)(\partial M'/\xi)\}+\Omega^2 z_0\xi(H'^2-2HH''-G^2)+\Omega^2 z_0(R_m/R_z)(2HM''\xi)-\Omega^2 z_0(R_L/R_z)(2LG/\xi)+(v_1\Omega/z_0)\{H'''\xi-(R_m/R_z)(M'''/\xi)\}-(2v_2/z_0)[n\Omega/2]\{(R_m/R_z)(\partial M'''/\xi)-\xi\partial H'''\}+\Omega^2\xi(H''^2-HH^{iv})+(R_m/R_z)(\Omega^2/\xi)(H'''M'+H''M''+H'M'''+HM^{iv})-(R_L/R_z)(2\Omega^2/\xi)(L'G'+LG'')]-(4v_3\Omega^2-z_0)\{(R_m/R_z)(1/2\xi)(H'''M'+H'M'''+H''M'')-(R_L/R_z)(1/2\xi)(2L'G'+LG'')+(\xi/4)(H''^2-G'^2-2H'H''')\}+(\sigma B_0^2\Omega z_0/p)\{-\xi H'+(R_m/R_z)(M'/\xi)\}.$$

(6.8)

$$0=n\Omega z_0\{\xi\partial G+(R_L/R_z)(\partial L/\xi)\}-(2\Omega^2 z_0\xi)(HG'-H'G)-\Omega^2 z_0(R_m/R_z)(2M'G/\xi)-\Omega^2 z_0(R_L/R_z)(2HL'/\xi)+(v_1\Omega/z_0)\{\xi G''+(R_L/R_z)(L''/\xi)\}+(2v_2/z_0)[(n\Omega/2)\{\xi\partial G''+(R_L/R_z)(\partial L''/\xi)\}+(R_L/R_z)(\Omega^2/\xi)(H''L'+H'''L+HL'''+H'L'')+(\Omega^2/\xi)(HG'''\ H''G')+(R_m/R_z)(2\Omega^2/\xi)(M'G''+M''G')]+(2v_3\Omega^2/z_0)\{\xi(H'G''-H''G')+(R_L+R_z)(1/\xi)(H''L'+H'''L+H'L'')+(R_m+R_z)(1/\xi)(2M''G'+M'G'')-(\sigma B_0^2\Omega z_0/p)\{\xi G+(R_L/R_z)(L/\xi)\}.$$

(6.9)

$-(1/pz_0)(\partial p/\partial \zeta)=2n\Omega z_0\partial H+4\Omega^2 z_0 HH'-2v_1\Omega H''/z_0-(2v_2/z_0)\ \{n\Omega\partial H''+2\Omega^2\xi$
$^2(H''H'''+G'G'')+\Omega^2(22H'HH''+2HH''')-$
$(R_m/R_z)2\Omega^2(H''M'''+H'''M'')+(R_L/R_z)\ 2\Omega^2\ (L'G''+L''G')\}-(2v_3$
$\Omega^2/z_0)\{\xi^2(H''H'''+G'G'')+14H'H''-(R_m/R_z)$
$(H''M'''+H'''M'')+(R_L/R_z)(L'G''+L''G')\}$
(6.10)

$pC_v(\partial T/\partial t+u\partial T/\partial r+w\partial T/\partial z)=K\{\partial^2T/\partial r^2+(1/r)\,\partial T/\partial r+\partial^2T/\partial z^2\}+\Phi$
(6.11)

where

$\Phi=\tilde{\tau}^i_j d^i_j$
(6.12)

C_v is the specific heat at constant volume, Φ be the viscous-dissipation function, $\tilde{\tau}^i_j$ is the mixed deviatoric stress tensor, K is the thermal conductivity, p is the density of the fluid; B and σ are intensity of the magnetic field and conductivity of the fluid considered.

Differentiating (6.8) w.r.t ζ and (6.10) w.r.t ζ and then eliminating $\partial^2 p/\partial\zeta.\partial\xi$ from the equation thus obtained. We get

$-n\Omega z_0\{\xi\partial H''-(R_m/R_z)\partial M''/\xi\}-2\Omega^2 z_0\xi(HH'''-GG')+(R_m/R_z)(2\Omega^2 z_0/\xi)$
$(H'M''+HM''')-(R_L/R_z)(2\Omega^2 z_0/\xi)(LG'+L'G)-(v_1\Omega/z_0)\{(R_m/R_z)(M^{iv}/$
$\xi)-\xi H^{iv}\}-(2v_2/z_0)\ [(n\Omega/2)\{(R_m/R_z)(\partial M^{iv}/\xi)-\xi\partial H^{iv}\}-\Omega^2\xi$
$(2H''H'''+H'H^{iv}+HH^{v}+4G'G'')+(R_m/R_z)(\Omega^2/\xi$
$)(2H'''M''+H^{iv}M'+2H''M'''+2H'M^{iv}+HM^{v})-(R_L/R_z)(2\Omega^2/\xi$
$)(2L'G''+L''G'+LG''')]-(2v_3\Omega^2/z_0)\{(R_m+R_z)(1/\xi)$
$(H^{iv}M'+2H'''M''+2H''M'''+H'M^{iv})-(R_L+R_z)(1/\xi$
$)(3L'G''+2L''G'+LG''')-\xi(H'H^{iv}+3G'G''+2H''H''')\}\ +\ (\sigma B$

$_0{}^2\Omega z_0/p)$ - $\{\xi H+(R_m/R_z)(M''/\xi)\}=0$

(6.13)

On equating the coefficients of ξ and $1/\xi$ from the equation (6.9) & (6.13), we get the following equations:

$$G''=R\partial G+2\in R(HG'-H'G)-\tau_1\partial G''-2\in\tau_1(HG'''-H''G')-2\in\tau_2(H'G''H''G')+m^2G$$

(6.14)

$$L''=R\partial L+2\in R(M'G+HL)-\tau_1\partial L''-2\in\tau_1(H''L'+H'''L+HL'''+H'L''+2M'G''+2M''G')-2\in\tau_2(H''L'+H'''L+H'L''+2M''G'+M'G')+m^2L$$

(6.15)

$$H^{iv}=R\partial H+2\in R(HH'''+GG')-\tau_1\partial H^{iv}-2\in\tau_1(H'H^{iv}+HH^{v}+2H''H'''+4G'G'')-2\in\tau_2(H'H^{iv}+2H''H''+3G'G'')+m^2H''$$

(6.16)

$$M^{iv}=R\partial M+2\in R(H'M''+HM'''-LG'-L'G)-\tau_1\partial M^{iv}-2\in\tau_1(2H'''M''+H^{iv}M'+2H''M'''+2H'M^{iv}+HM^{v}-4L'G''-2L''G'-2LG''')-2\in\tau_2(H^{iv}M'+2H'''M''+2H''M'''+H'M^{iv}-3L'G''-2L''G'-LG''')+m^2M''$$

(6.17)

where $R(=nz_0/v_1)$ is the Reynolds number, $\tau_1(=nv_2/v_1)$, $\tau_2(=nv_3/v_1)$ and $\in(=\Omega/n)$ are the dimensionless parameter, $m^2=\sigma B_0{}^2z_0{}^2/\mu_1$ is the dimensionless magnetic field and $R_m/R_L=\mathbf{m/L}\approx 1$.

Using (6.3), (6.12) in (6.11) and equating the coefficient of ξ^2, we get

$$\Psi'' = \in RP_r[\partial\Psi/\in -2H'\Psi+2H\Psi'-H''^2+G'^2-\tau_1(H''\partial H''+G'\partial G')-2$$
$$\in \tau_1 (H'H''^2+H'G'^2HH''H'''+HG'G'')-3 \in \tau_2(H'H''^2+H'G'^2)],$$

$$4\Psi+\phi'' = \in RP_r[\partial\phi/\in +(R_m/R_z)2M'\Psi+2H\phi'-$$
$$12H'^2+(R_m/R_z)2H''M''-(R_L/R_z)2L'G'-\tau_1\{12H'\partial H'-(R_m/R_z)(H''\partial$$
$$M''+M''\partial H'')+(R_L/R_z)(G'\partial L'+L'\partial L')\} -2 \in \tau$$
$$_1\{(12H'^3+12HH'H''+(R_m/R_z)(2M'G'^2-HH'''M''-H''^2M'-$$
$$HH''M''')+(R_L/R_z)(HL'G'+3H'L'G'+3LG'H''+HG'L'')\} - \in \tau$$
$$_2\{(R_m/R_z)(3M'G'^2-6H'H''M)+24H'^3+6(R_L/R_z)(H''LG'+H'L'G')\}]$$

Where $P_r=\mu_1 C_v/K$ is the Prandtl number.

For $R_m=R_L=B_0=0$, the differential equations (6.8)-(6.10) are identical to those obtained by Sharma & Gupta[67] (for $S_1=1$, $S_2=0$), Sharma & Singh[68] (for $S_1=1$, $S_2=0$) and for $R_m=R_L=B_0=0$,the differential equations (6.8)-(6.10),(6.18), (6.19) are identical to those obtained by Sharma & Singh[79] (for $S_1=1$, $S_2=0$).

VI.3 SOLUTION OF THE PROBLEM

Substituting the expressions

$$G(\zeta,\tau) = \sum \in^N G_N(\zeta,\tau)$$

$$L(\zeta,\tau) = \sum \in^N L_N(\zeta,\tau)$$

$$H(\zeta,\tau) = \sum \in^N H_N(\zeta,\tau)$$

$$M(\zeta,\tau) = \sum \in^N M_N(\zeta,\tau)$$

$$\phi(\zeta,\tau) = \sum \in^N \phi_N(\zeta,\tau)$$

$$\Psi(\zeta,\tau) = \sum \in^N \Psi_N(\zeta,\tau)$$

(6.20)

into (6.14) to (6.19) neglecting the terms with coefficient of $\in^2$ (assumed negligible small) and equating the terms independent of $\in$ and coefficient of $\in$, we get the following equations:

$G_0'' = R\,\partial G_0/\partial\tau - \tau_1\,\partial G_0''/\partial\tau + m^2 G_0$

(6.21)

$G_1'' = R\,\partial G_1/\partial\tau - 2R(H_0' G_0' - H_0 G_0') - \tau_1\,\partial G_1''/\partial\tau - 2\tau_1(H_0' G_0''' - H_0'' G_0') - 2\tau_2(H_0' G_0'' -$

$H_0'' G_0') + m^2 G_1$

(6.22)

$L_0'' = R\,\partial L_0/\partial\tau - \tau_1\,\partial L_0''/\partial\tau + m^2 L_0$

(6.23)

$L_1'' = R\,\partial L_1/\partial\tau - 2R(M_0' G_0' - H_0 L_0') - \tau_1\,\partial L_1''/\partial\tau - 2\tau_1(H_0''' L_0 + H_0'' L_0' + H_0' L_0'' + H_0 L_0'''$

$+2M_0'' G_0' + 2M_0' G_0'') - 2\tau_2(H_0''' L_0 + H_0'' L_0' + H_0' L_0'' + 2M_0'' G_0' + M_0' G_0'') + m^2 L_1$

(6.24)

$H_0^{iv} = R\partial H_0''/\partial\tau - \tau_1\partial H_0^{iv}/\partial\tau + m^2 H_0''$

(6.25)

$H_1^{iv} = R\,\partial H_1''/\partial\tau + 2R(H_0 H_0''' + G_0 G_0') - \tau_1\,\partial H_1^{iv}/\partial\tau - 2\tau_1(H_0' H_0^{iv} + H_0 H_0^{v} + 2H_0'' H_0''' +$

$4G_0' G_0'') - 2\tau_2(3G_0' G_0'' + H_0' H_0^{iv} + 2H_0'' H_0''') + m^2 H_1''$

(6.26)

$M_0^{iv} = R\partial M_0''/\partial\tau - \tau_1\partial M_0^{iv}/\partial\tau + m^2 M_0''$

(6.27)

$M_1^{iv}=R\partial M_1''/\partial\tau-2R(H_0'M_0''+H_0M_0'''-L_0'G_0-L_0G_0')-\tau_1\partial M_1^{iv}/\partial\tau-2\tau_1(2H_0'''M_0''$

$+H_0^{iv}M_0'+2H_0''M_0'''-4L_0'G_0''-2L_0''G_0'-2L_0G_0''')+H_0M_0^{v}+2H_0'M_0^{iv})-2\tau_2(2H_0'''M_0''+H_0^{iv}M_0'+2H_0''M_0'''-3L_0'G_0''-2L_0''G_0'-L_0G_0'''+H_0'M_0^{iv})+m^2M_1$ (6.28)

$\Psi_0''=RP_1\partial\Psi_0,$ (6.29)

$\Psi_1''=RP_r[\partial\Psi_1-2H_0'\Psi_0+2H_0\Psi_0'-H_0''^2-G_0'^2-\tau_1(H_0''\partial H_0''+G_0'\partial G_0'),$ (6.30)

$4\Psi_0+\phi_0''=RP_1\partial\Psi_0,$ (6.31)

$4\Psi_1+\phi_0''=RP_r[\partial\phi_1+(R_m/R_z)2M_0'\Psi_0+2H_0\phi_0'-12H_0'^2+(R_m/R_z)2H_0''M_0''-(R_L/R_z)2L_0'G_0'-\tau_1\{(12H_0'\partial H_0'-(R_m/R_z)(H_0''\partial M_0''+M_0''\partial H_0'')+(R_L/R_z)(G_0'\partial L_0')\}].$ (6.32)

Taking $G_n(\zeta,\tau) = G_{ns}(\zeta)+e^{i\tau}G_{nt}(\zeta)$

$L_n(\zeta,\tau) = L_{ns}(\zeta)+e^{i\tau}L_{nt}(\zeta)$

$H_n(\zeta,\tau) = H_{ns}(\zeta)+e^{2i\tau}H_{nt}(\zeta)$

$M_n(\zeta,\tau) = M_{ns}(\zeta)+e^{2i\tau}M_{nt}(\zeta)$

$\phi_n(\zeta,\tau) = \phi_{ns}(\zeta)+e^{2i\tau}\phi_{nt}(\zeta)$

$\Psi_n(\zeta,\tau) = \Psi_{ns}(\zeta)+e^{2i\tau}\Psi_{nt}(\zeta)$

(6.33)

Complex notation has been adopted here with the convention that only real pats of the complex quantities have the physical meaning.

Using (6.24) and (6.33), the boundary conditions (6.5) & (6.7) for n = 0,1 transform to

$$G_{0s}(0) = 0, \quad G_{0t}(0) = 1, \quad G_{1s}(0) = 0, \quad G_{1t}(0) = 0,$$

$$G_{0s}(1) = 0, \quad G_{0t}(0) = 0, \quad G_{1s}(1) = 0, \quad G_{1t}(1) = 0,$$

$$H_{0s}(0) = k, \quad H_{0t}(0) = 0, \quad H_{1s}(0) = 0, \quad H_{1t}(0) = 0,$$

$$H_{0s}(1) = k, \quad H_{0t}(1) = 0, \quad H_{1s}(1) = 0, \quad H_{1t}(1) = 0,$$

$$H'_{0s}(0) = 0, \quad H'_{0t}(0) = 0, \quad H'_{1s}(0) = 0, \quad H'_{1t}(0) = 0,$$

$$H'_{0s}(1) = 0, \quad H'_{0t}(1) = 0, \quad H'_{1s}(1) = 0, \quad H'_{1t}(1) = 0,$$

$$L_{0s}(0) = 0, \quad L_{0t}(0) = 0, \quad L_{1s}(0) = 0, \quad L_{1t}(0) = 0,$$

$$L_{0s}(1) = 0, \quad L_{0t}(1) = 0, \quad L_{1s}(1) = 0, \quad L_{1t}(1) = 0,$$

$$M'_{0s}(0) = 0, \quad M'_{0t}(0) = 0, \quad M'_{1s}(0) = 0, \quad M'_{1t}(0) = 0,$$

$$M'_{0s}(1) = 0, \quad M'_{0t}(1) = 0, \quad M'_{1s}(1) = 0, \quad M'_{1t}(1) = 0,$$

$$M_{0s}(0) = 0, \quad M_{0t}(0) = 0, \quad M_{1s}(0) = 0, \quad M_{1t}(0) = 0,$$

$$M_{0s}(0) = 0, \quad M_{0t}(0) = 0, \quad M_{1s}(0) = 0, \quad M_{1t}(0) = 0,$$

$$M_{0s}(1) = 0, \quad M_{0t}(1) = 0, \quad M_{1s}(1) = 0, \quad M_{1t}(1) = 0,$$

$$\Psi_{0s}(0) = 0, \quad \Psi_{0t}(0) = 0, \quad \Psi_{1s}(0) = 0, \quad \Psi_{1t}(0) = 0,$$

$$\Psi_{0s}(1) = 0, \quad \Psi_{0t}(1) = 0, \quad \Psi_{1s}(1) = 0, \quad \Psi_{1t}(1) = 0,$$

$$\phi_{0s}(0) = 0, \quad \phi_{0t}(0) = 0, \quad \phi_{1s}(0) = 0, \quad \phi_{1t}(0) = 0,$$

$$\phi_{0s}(1) = 0, \quad \phi_{0t}(1) = 0, \quad \phi_{1s}(1) = 0, \quad \phi_{1t}(1) = 0, \qquad (6.34)$$

Applying (6.33) & (6.34) in eqs. (6.21) to (6.32), we get

$G_{0s}(\zeta) = G1s(\zeta) = 0,$

$G_{0t}(\zeta) = \{1-(e^{f}/2\text{Sinh f})\}e^{f\zeta}+(e^{f}/2\text{Sinh f})\, e^{-f\zeta},$

where $f = \{(iR+m^2)/(1+i\tau)\}^{1/2}= A+iB,$

where $A = [[(m^2+R\tau_1)^2+(m^2+R\tau_1)^2+(R-m^2\tau_1)^2\}^{1/2}/\{2(1+\tau_1{}^2\}]^{1/2},$

$B = [[(m^2+R\tau_1)^2+(R-m^2\tau_1)^2\}^{1/2}-(m^2 +R\tau_1)]/\{2(1+\tau_1{}^2\}]^{1/2},$

$G_0(\zeta,\tau) = \text{Real}\{e^{i\tau}G_{0t}(\zeta)\},$

$= (A_3+A_5)\text{Cos}\tau-(A_4+A_6)\text{Sin}\tau,$

where

$A_1=e^{A}(\text{Cos}^2B\text{Sinh}A+ \text{Cosh}A\text{Sin}^2B)/2(\text{Sinh}^2A+\text{Sin}^2B)\},$

$A_2=e^{A}(\text{Sin}B\text{Cos}B\text{Sinh}A-\text{Cos}B\text{Cosh}A\text{Sin}B)/2(\text{Sinh}^2A+\text{Sin}^2B)\},$

$A_3=e^{A\zeta}\{(1-A_1)\text{Cos}B\zeta+A_2\text{Sin}B\zeta\},$

$A_4=e^{A\zeta}\{(1-A_1)\text{Sin}B\zeta+A_2\text{Cos}B\zeta\},$

$A_5=e^{-A\zeta}\{A_1\text{Cos}B\zeta+A_2\text{Sin}B\zeta\},$

$A_6=e^{-A\zeta}\{A_2\text{Cos}B\zeta-A_1\text{Sin}B\zeta\},$

$G_{1\tau}(\zeta) = C_1e^{f\zeta}+ C_2e^{-f\zeta}+Rk[a_1e^{f\zeta}\{\zeta-(1/2f)+ae^{-f\zeta}\{\zeta-(1/2f)\}],$

Where $a_1=1-(e^{f}/2\text{Sinh}f),$

$a_2= e^{f}/2\text{Sinh}f,$

$C_1=-C\{C_2+(Rk/2f)(a_2-a_1)\},$

$C_2=-(Rk/2\text{Sinh}f)(a_1e^{f}+a_2e^{-f}-(a_2\text{Sinh}f/f)\},$

$G(\zeta,\tau)= G_0(\zeta,\tau)+\in G_1(\zeta,\tau).$

$M_{0t}(\zeta)= M_{1t}(\zeta)=0,$

$M_{0s}(\zeta) = A_1 e^{m\zeta}/m^2+ A_2 e^{-m\zeta}/m^2+A_3\zeta+A_{4,}$

$M'_{0s}(\zeta) = A_1 e^{m\zeta}/m-A_2 e^{-m\zeta}/m^2+A_3$

Where $A_1 = (e^{-m}-1)/ (e^{m}-1)_,$

$A_2 =m^2(e^m-1)/\{(e^m(m-2)-e^{-m}(m+2)\},$

$A_3 = -(A_1- A_2)/m,$

$A_4 = (A_1+ A_2)/m^2.$

$M_{1s}(\zeta)= C_5 e^{m\zeta}/m^2+ C_6 e^{-m\zeta}/m^2+(Rk-\tau_1 km^2)[C_1 e^{m\zeta}(2m\zeta-5)/2m^3+$

$C_5 e^{-m\zeta}(2m\zeta+5)/2m^3]+C_7\zeta+C_{8,}$

where $C_5 = \{m^2(e^{-m}-1)(D_1+D_2-D_3)+m(e^{-m}+m-1)(D_2-D_4\}/4-e^{-m}(m+2)+e^m(m-2)\},$

$C_6 = \{C_5(e^m-1)-m(D_3-D_4)\}/(e^{-m}-1),$

$C_7 = -(C_5/m)+(C_6/m)-D_{2,}$

$C_8 = -(C_5/m^2)-(C_6/m^2)-D_1$

$D_1 = (Rk-\tau_1 km^3)(5/2m^3)(C_2-C_1),$

$D_2 = -(Rk-\tau_1 km^2)(3/2m^3)(C_1+C_2),$

$D_3 = -(Rk-\tau_1 km^2)(1/2m^3)\{C_1 e^m(2m-5)+ C_2 e^{-m}(2m+5)\},$

$D_4 = (Rk-\tau_1 km^2)(1/2m^2)\{C_1 e^m(2m-3)- C_2 e^{-m}(2m+3)\},$

$M(\zeta,\tau)= M_0(\zeta,\tau)+\in M_1(\zeta,\tau).$

$H_{0t}(\zeta)= H_{1t}(\zeta) = 0,$

$H_{0s}(\zeta) = k$

$H_{1s}(\zeta) = C_1 e^{d\zeta}/d^2 + C_2 e^{-d\zeta}/d^2 + C_3\zeta + C_4 + (Z_1 - f^2 Z_2)[\{1-(e^f/2Sinhf\}^2 e^{2f\zeta}/(16fSinh^2 f)]$

where $d = \{2iR+m^2)/(1+2i\tau)\}^{1/2} = C+iD$,

where $C=[[m^2+4R\tau_1)+\{ [m^2+4R\tau_1)^2+[2R-2m^2\tau_1)^2\}^{1/2}]/\{2(1+4\tau_1{}^2)\}]^{1/2}$,

$D=[[m^2+4R\tau_1)^2+\{ [2R-2m^2\tau_1)^2\}^{1/2}-[m^2+4R\tau_1)]/\{2(1+4\tau_1{}^2)\}]^{1/2}$

$Z_1 = 2R/\{(1+i\tau_1)(4f^2-d^2)\}$,

$Z_2 = (8\tau_1+6\tau_2)/\{(1+2i\tau_i)(4f^2-d^2)\}$,

$C_1=\{d(Z_{10}- Z_9-Z_6+Z_5+C_2(e^{-d}-1)\}$,

$C_2=Z_{11}/\{4-e^{-d}(d+2)+e^d(d-2)\}$,

$C_3=-\{(C_1/d)- (C_2/d)+Z_5-Z_6\}$,

$C_4=-\{(C_1/d^2)+(C_2/d^2)+Z_3-Z_4\}$,

$Z_3 = Z_1[\{1-(e^f/2Sinhf)\}^2/4-fe^{2f}/16fSinh^2 f]$,

$Z_4 = Z_2[f\{1-(e^f/2Sinhf)\}^2/4-fe^{2f}/16fSinh^2 f]$,

$Z_5 = Z_1[\{1-(e^f/2Sinhf)\}^2/2+e^{2f}/8Sinh^2 f]$,

$Z_6 = Z_2[f^2\{1-e^f/2Sinhf)\}^2/2+f^2e^{2f}/8Sinh^2 f]$,

$Z_7 = Z_1[e^{2f}\{1-e^f/2Sinhf)\}^2/4f-1/16fSinh^2 f]$,

$Z_8 = Z_2[fe^{2f}\{1-e^f/2Sinhf)\}^2/4-f/16fSinh^2 f]$,

$Z_9 = Z_1[e^{2f}\{1-e^f/2Sinhf)\}^2/2+1/8Sinh^2 f]$,

$Z_{10} = Z_2[f^2e^{2f}\{1-e^f/2Sinhf)\}^2/2-f^2/8fSinh^2 f]$,

$Z_{11} = d^2(e^d-1)(Z_3-Z_4+Z_5-Z_6-Z_7+Z_8)-d(e^d-d-1)\ Z_{10}-Z_9-Z_6+Z_5)$,

$H_1(\zeta,\tau)$ = Real$\{e^{2i\tau}H_{1t}(\zeta)\}$,

$= (A_{81}+A_{89})\text{Cos}2\tau-(A_{82}+A_{90})\text{Sin}2\tau,$

where

$A_{13} = 4A^2-4B^2-C^2+D^2,$

$A_{14} = 8AB-2CD,$

$A_{15}= 2R(A_{13}-\tau_1A_{14})/\{ (A_{13}-\tau_1A_{14})^2+(\tau_1 A_{13}+ A_{14})^2\},$

$A_{16}= -2R(A_{13}\tau_1+A_{14})/\{ (A_{13}-\tau_1A_{14})^2+(\tau_1 A_{13}+ A_{14})^2\},$

$A_{17}= (8\tau_1 +6\tau_2)(A_{13}-2\tau_1A_{14})/\{(A_{13}-2\tau_1A_{14})^2+(2\tau_1A_{13}+ A_{14})^2\},$

$A_{18}= -(8\tau_1 +6\tau_2)(2\tau_1A_{13}+ A_{14})/\{(A_{13}-2\tau_1A_{14})^2+(2\tau_1A_{13}+ A_{14})^2\},$

$A_{19}=[A\{(1-A_1)^2-A_2^2\}+2BA_2(1-A_1)]/\{4(A^2+B^2)\},$

$A_{20}=-[B\{(1-A_1)^2-A_2^2\}+2AA_2(1-A_1)]/\{4(A^2+B^2)\},$

$A_{21}=\text{Sinh}^2A\text{Cos}^2B-\text{Cosh}^2A\text{Sin}^2B,$

$A_{22}=2\text{Sinh}A\text{Cos}B\text{Cosh}A\text{Sin}B,$

$A_{23}= e^{2A} (A_{21}\text{Cos}2B+A_{22}\text{Sin}2B)/(A_{21}^2+A_{22}^2),$

$A_{24}= e^{2A} (A_{21}\text{Sin}2B-A_{22}\text{Cos}2B)/(A_{21}^2+A_{22}^2),$

$A_{25}= (AA_{23}+BA_{24})/\{16(A^2+B^2)\},$

$A_{26}= (AA_{24}-BA_{23})/\{16(A^2+B^2)\},$

$A_{27}=A_{15} (A_{19}-A_{25})-A_{16}(A_{20}+A_{26}),$

$A_{28}=A_{16} (A_{19}-A_{25})+A_{15}(A_{20}-A_{26}),$

$B_1=(1-A_1)^2A_2^2,$

B_2=-2(1-A_1)A_2,

$X_1 = (A^2-B^2)(A_{17}A_{27}-A_{18}A_{28})-2AB(A_{18}A_{27}+A_{17}A_{28})$,

$X_2 = 2AB(A_{17}A_{27}-A_{18}A_{28})+ (A^2-B^2)(A_{18}A_{27}+A_{17}A_{28})$,

$A_{29}=(X_1A_{15}+X_2A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{30}=(X_2A_{15}-X_1A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{31}=A_{15}\{(B_1/2)+(A_{23}/8)\}-A_{16}\{(B_2/2)+(A_{24}/8)\}$,

$A_{32}=A_{16}\{(B_1/2)+(A_{23}/8)\}-A_{15}\{(B_2/2)+(A_{24}/8)\}$,

$X_3 = (A^2-B^2)(A_{17}A_{31}-A_{18}A_{32})-2AB(A_{18}A_{31}+A_{17}A_{32})$,

$X_4 = 2AB(A_{17}A_{31}-A_{18}A_{32})+ (A^2-B^2)(A_{18}A_{31}+A_{17}A_{32})$,

$A_{33}=(X_3A_{15}+X_4A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{34}=(X_4A_{15}-X_3A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{35}=e^{2A}(A_{19}Cos2B-A_{20}Sin2B)$,

$A_{36}=e^{2A}(A_{20}Cos2B+A_{19}Sin2B)$,

$A_{37}=(AA_{21}-BA_{22})/[16\{(AA_{21}-BA_{22})^2+(BA_{21}-AA_{22})^2\}]$,

$A_{38}=-(BA_{21}+AA_{22})/[16\{(AA_{21}-BA_{22})^2+(BA_{21}-AA_{22})^2\}]$,

$A_{39}=A_{15}(A_{35}-A_{37})-A_{16}(A_{36}-A_{38})$,

$A_{40}=A_{16}(A_{35}-A_{37})-A_{15}(A_{36}-A_{38})$,

$X_5 = (A^2-B^2)(A_{17}A_{39}-A_{18}A_{40})-2AB(A_{18}A_{39}+A_{17}A_{40})$,

$X_6 = 2AB (A_{17}A_{39}-A_{18}A_{40})+ (A^2-B^2) (A_{18}A_{39}+A_{17}A_{40})$,

$A_{41}=(X_5A_{15}+X_6A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{42}=(X_6A_{15}-X_5A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{43}=e^{2A}(B_1Cos2B-B_2Sin2B)/2$,

$A_{44}=e^{2A}(B_2Cos2B+B_1Sin2B)/2$,

$A_{45}=A_{21}/\{8(A_{21}^2+A_{22}^2)\}$,

$A_{46}=-A_{22}/\{8(A_{21}^2+A_{22}^2)\}$,

$A_{47}=A_{15}(A_{43}+A_{45})-A_{16}(A_{44}+A_{46})$,

$A_{48}=A_{16}(A_{43}+A_{45})-A_{15}(A_{44}+A_{46})$,

$X_7 = (A^2-B^2)(A_{17}A_{47}-A_{18}A_{48})-2AB(A_{18}A_{47} +A_{17}A_{48})$,

$X_8 = 2AB (A_{17}A_{47}-A_{18}A_{48})+ (A^2-B^2) (A_{18}A_{47} +A_{17}A_{48})$,

$A_{49}=(X_7A_{15}+X_8A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{50}=(X_8A_{15}-X_7A_{16})/(A_{15}^2+A_{16}^2)$,

$A_{51}=(C^2-D^2)(e^cCosD-1)-2CDe^CSinD$,

$A_{52}=2CD(e^cCosD-1)- (C^2-D^2)e^CSinD$,

$A_{53}=C(e^cCosD-C-1)- D(e^CSinD-D)$,

$A_{54}=D(e^cCosD-C-1)- C(e^CSinD-D)$,

$A_{55}= A_{27}-A_{29}+A_{31}-A_{33}-A_{39}+A_{41}$,

$A_{56}= A_{28}-A_{30}+A_{32}-A_{34}-A_{40}+A_{42}$,

$A_{57}= A_{49}-A_{47}-A_{33}+A_{31}$,

$A_{58}= A_{50}-A_{48}-A_{34}+A_{32}$,

$A_{59}= A_{51}A_{55}-A_{52}A_{56}- A_{53}A_{57}+A_{54}A_{58}$,

$A_{60}= A_{52}A_{55}-A_{51}A_{56}- A_{54}A_{57}+A_{53}A_{58}$,

$A_{65}=4-e^{-C}\{(C+2)CosD+DSinD\}+e^{c}\{(C-2) CosD-DSinD\}$,

$A_{66}=-e^{-C}\{DCosD-(C+2)DSinD\}+e^{c}\{(C-2)SinD+DCosD\}$,

$A_{67}=(A_{59}A_{65}+A_{60}A_{66})/(A_{65}^{2}+A_{66}^{2})$,

$A_{68}=(A_{60}A_{65}-A_{59}A_{66})/(A_{65}^{2}+A_{66}^{2})$,

$A_{69}=CA_{57}-DA_{58}+A_{67}(e^{-c}cosD-1)+ A_{68}e^{-c}SinD$,

$A_{70}=DA_{57}+CA_{58}+A_{67}(e^{-c}cosD-1)+ A_{68}e^{-c}SinD$

$A_{71}=\{A_{69}(e^{c}cosD-1)+A_{70}\ e^{c}SinD\}/\{(e^{c}cosD-1)^{2}+e^{2c}Sin^{2}D\}$,

$A_{72}=\{A_{70}(e^{c}cosD-1)-A_{69}\ e^{c}SinD\}/\{(e^{c}cosD-1)^{2}+e^{2c}Sin^{2}D\}$,

$A_{73}=-[\{(CA_{71}+DA_{72}-CA_{67}-DA_{68})/(C^{2}+D^{2})\}+A_{31}-A_{33}]$,

$A_{74}=-[\{(CA_{72}-DA_{71}-CA_{68}-DA_{67})/(C^{2}+D^{2})\}+A_{32}-A_{3}]$,

$A_{75}=\{A_{71}(C^{2}-D^{2})+2CDA_{72}\}/\{(C^{2}-D^{2})+4C^{2}D^{2}\}$,

$A_{76}=\{A_{72}(C^{2}-D^{2})-2CDA_{71}\}/\{(C^{2}-D^{2})+4C^{2}D^{2}\}$,

$A_{77}=\{A_{67}(C^{2}-D^{2})+2CDA_{68}\}/\{(C^{2}-D^{2})+4C^{2}D^{2}\}$,

$A_{78}=\{A_{68}(C^{2}-D^{2})+2CDA_{67}\}/\{(C^{2}-D^{2})+4C^{2}D^{2}\}$,

$A_{79}=-(A_{75}+A_{77}+A_{27}-A_{29})$,

$A_{80}=-(A_{76}+A_{78}+A_{28}-A_{30})$,

$A_{81}=e^{C\zeta}(A_{75}CosD\zeta-A_{76}SinD\zeta)+ e^{-C\zeta}(A_{77}CosD\zeta+A_{78}SinD\zeta)+A_{73}\zeta+A_{79}$,

$A_{82}=e^{C\zeta}(A_{76}CosD\zeta+A_{75}SinD\zeta)+ e^{-C\zeta}(A_{78}CosD\zeta-A_{77}SinD\zeta)+A_{74}\zeta+A_{80}$,

$A_{83}= A_{15}-(A^{2}-B^{2})A_{17}+2ABA_{18}$,

$A_{84}= A_{16}-(A^2-B^2)A_{18}-2ABA_{17}$,

$A_{85}=e^{2A\zeta}(A_{19}Cos2B\zeta-A_{20}Sin2B\zeta)$,

$A_{86}=e^{2A\zeta}(A_{20}Cos2B\zeta+A_{19}Sin2B\zeta)$,

$A_{87}=e^{2A(1-\zeta)}(A_{37}Cos2B(1-\zeta)-A_{38}Sin2B(1-\zeta)\}$,

$A_{88}=e^{2A(1-\zeta)}(A_{38}Cos2B(1-\zeta)+A_{37}Sin2B(1-\zeta)\}$,

$A_{89}=A_{83}(A_{85}-A_{87})-A_{84}(A_{86}-A_{88})$,

$A_{90}=A_{84}(A_{85}-A_{87})+A_{83}(A_{86}-A_{88})$,

$H(\zeta,\tau)=H_0(\zeta,\tau)+\in H_1(\zeta,\tau)= \in H_1(\zeta,\tau)$

$L_{0s}(\zeta)=L_{0t}(\zeta)= L_{1s}(\zeta) = 0$

$L_{1t}(\zeta) = C_5e^{f\zeta}+ C_6e^{-f\zeta}+\{a_3e^{(m+f)\zeta}+ a_7e^{-(m+f)\zeta}\}/(m^2+2mf)+ \{a_4e^{-(m+f)\zeta}+ a_6e^{(m+f)\zeta}\}/ (m^2-2mf)+(\zeta/2f) \{a_5e^{f\zeta}+ a_8e^{-f\zeta})- \{a_5e^{f\zeta}+ a_8e^{-f\zeta})/4f^2$,

Where

$C_5=-[C_6+\{(a_{3+}a_7)/m^2+2mf)\}+\{(a_4+a_6)/(m^2-2mf)\}-\{(a_5+a_8)/(4f^2)\}]$,

$C_6=[[a_3\{e^{(m+f)}-e^f\}+\{a_7e^{-(m+f)}-e^f\}]/(m^2+2mf)+[a_4\{e^{-(m-f)}-e^f\}+a_6\{e^{(m-f)}-e^f\}](m^2-2mf)+(a_5e^f-a_8e^{-f})/2f+a_8Sinhf/2f^2]/(2Sinhf)$,

$a_1=1-(e^f/2Sinhf)$,

$a_2=e^f/2Sinhf$,

$a_3=\{2RC_1a_1/m(1+i\tau_1)\}-\{4(\tau_1+\tau_2)C_1a_1f/(1+i\tau_1\})-\{(4\tau_1+2\tau_2)C_1a_1f^2/m(1+i\tau_1\})$,

$a_4=-\{2RC_2a_1/m(1+i\tau_1)\}-\{4(\tau_1+\tau_2)C_2a_1f/(1+i\tau_1\})-\{(4\tau_1+2\tau_2)C_2a_1f^2/m(1+i\tau_1\})$,

$a_5=\{2RC_3a_1/(1+i\tau_1)\}-\{4\tau_1+2\tau_2)C_2a_1f^2/(1+i\tau_1)$,

$a_6=\{2RC_1a_2/m(1+i\tau_1)\}-\{4(\tau_1+\tau_2)C_1a_2f/(1+i\tau_1\})-$
$\{(4\tau_1+2\tau_2)C_1a_2f^2/m(1+i\tau_1\})$,

$a_7=-\{2RC_2a_2/m(1+i\tau_1)\}-\{4(\tau_1+\tau_2)C_2a_2f/(1+i\tau_1\})-$
$\{(4\tau_1+2\tau_2)C_2a_2f^2/m(1+i\tau_1\})$,

$a_8=\{2RC_3a_2/(1+i\tau_1)\}-\{4\tau_1+2\tau_2)C_3a_2f^2/(1+i\tau_1\})$,

$L_1(\zeta,\tau)=\text{Real}\{e^{i\tau}L_{1t}(\zeta)\}$,

$= (B_{47}+B_{51}+B_{55}+B_{59}-B_{63})\text{Cos}\tau-(B_{48}+B_{52}+B_{56}+B_{60}-B_{64})\text{Sin}\tau$,

$B_3=1/(1+\tau_1^2)$,

$B_4=-\tau_1/(1+\tau_1^2)$,

$B_5=(2RC_1/m)\{B_3(1-A_1)+B_4A_2\}-4(\tau_1+\tau_2)C_1\{(1-A_1)(AB_3-$
$BB_4)+A_2(AB_4+BB_3)\}-(4\tau_1+2\tau_2)C_1/m\}(A^2-B^2)\{B_3(1-A_1)+B_4A_2\}-$
$2AB\{B_4(1-A_1)- B_3A_2\}]$,

$B_6=(2RC_1/m)\{B_4(1-A_1)-B_3A_2\}-4(\tau_1+\tau_2)C_1\{(1-A_1)(AB_4-BB_3)+A_2(AB_3-$
$BB_4)\}-(4\tau_1+2\tau_2)C_1/m\}[2AB\{B_3(1-A_1)+B_4A_2\}+(A^2-B^2)\{B_4(1-A_1)-$
$B_3A_2\}]$,

$B_7=-(2RC_2/m)\{B_3(1-A_1)+B_4A_2\}-4(\tau_1+\tau_2)C_2\{(1-A_1)(AB_3-$
$BB_4)+A_2(AB_4+BB_3)\}+ (4\tau_1+2\tau_2)C_2/m\}(A^2-B^2)\{B_3(1-A_1)+B_4A_2\}-$
$2AB\{B_4(1-A_1)- B_3A_2\}]$,

$B_8=-(2RC_2/m)\{B_4(1-A_1)-B_3A_2\}-4(\tau_1+\tau_2)C_2\{(1-A_1)(AB_4+BB_3)+A_2(AB_3-$
$BB_4)\}+ \{(4\tau_1+2\tau_2)C_2/m\}[2AB\{B_3(1-A_1)+B_4A_2\}+(A^2-B^2)\{B_4(1-A_1)-$
$B_3A_2\}]$,

B_9=2RC_3{B_3(1-A_1)+B_4A_2}-4(τ_1+2τ_2)C_3[(A^2-B^2){B_3(1-A_1)+B_4A_2}-(2AB){B_4(1-A_1)-

B_3A_2}],

B_{10}=2RC_3{B_4(1-A_1)+B_3A_2}-4(τ_1+2τ_2)C_3[2AB){B_3(1-A_1)+B_4A_2}+(A^2-B^2){B_4(1-A_1)-

B_3A_2}],

B_{11}=(2RC_1/m){B_3A_1-B_4A_2}+4(τ_1+τ_2)C_1{A{B_3A_1-B_4A_2}-B(B_4A_1+B_3A_2)}-{(4τ_1+2τ_2)C_1/m}[{B_3A_1-B_4A_2)(A^2-B^2)-2AB{B_4A_1+B_3A_2}],

B_{12}=(2RC_1/m){B_4A_1+B_3A_2}+4(τ_1+τ_2)C_1{B{B_3A_1-B_4A_2}+A(B_4A_1+B_3A_2)}- {(4τ_1+2τ_2)C_1/m}[{B_3A_1-B_4A_2)2AB+(A^2-B^2)(B_4A_1+B_3A_2}],

B_{13}=-(2RC_2/m){B_3A_1-B_4A_2}+4(τ_1+τ_2)C_2{A{B_3A_1-B_4A_2}-B(B_4A_1+B_3A_2)}+ {(4τ_1+2τ_2)C_2/m}[{B_3A_1-B_4A_2)(A^2-B^2)-2AB{B_4A_1+B_3A_2}],

B_{14}=-(2RC_2/m){B_4A_1+B_3A_2}+4(τ_1+τ_2)C_2{B{B_3A_1-B_4A_2}+A(B_4A_1+B_3A_2)}- {(4τ_1+2τ_2)C_2/m}[{B_3A_1-B_4A_2)2AB+(A^2-B^2)(B_4A_1+B_3A_2}],

B_{15}=2RC_3{B_3A_1-B_4A_2}-4(τ_1+τ_2)C_3{(A^2-B^2)(B_3A_1-B_4A_2}-2AB(B_4A_1+B_3A_2)],

B_{16}=2RC_3{B_4A_1+B_3A_2}-4(τ_1+τ_2)C_3{(B_3A_1-B_4A_2}2AB+(A^2-B^2)(B_4A_1+B_3A_2)],

Y_1=B_5CosB($e^{(m+A)}$-e^A)-B_6SinB($e^{(m+A)}$-e^A),

Y_2=B_6CosB($e^{(m+A)}$-e^A)+B_5SinB($e^{(m+A)}$-e^A),

$Y_3=B_{13}CosB(e^{-(m+A)}-e^A)+B_{14}SinB(e^{-(m+A)}+e^A)$,

$Y_4=B_{14}CosB(e^{-(m+A)}-e^A)-B_{13}SinB(e^{-(m+A)}+e^A)$,

$B_{17}=(Y_1+Y_3)$,

$B_{18}=(Y_2+Y_4)$,

$Y_5=(e^{-(m-A)}-e^A)(B_7CosB- B_8SinB)$,

$Y_6=(e^{-(m-A)}-e^A)(B_8CosB- B_7SinB)$,

$Y_7= B_{11}CosB(e^{(m-A)}-e^A)+ B_{12}SinB(e^{(m-A)}+e^A)$

$Y_8= B_{12}CosB (e^{(m-A)}-e^A)-B_{11}SinB(e^{(m-A)}+e^A)$,

$B_{19}=(Y_5+Y_7)$,

$B_{20}=(Y_6+Y_8)$,

$B_{21}=(m^2+2mA)/\{(m^2+2mA)^2+4m^2B^2\}$,

$B_{22}=-2mB/\{(m^2+2mA)^2+4m^2B^2\}$,

$B_{23}=(m^2-2mA)/\{(m^2-2mA)^2+4m^2B^2\}$,

$B_{24}=2mB/\{(m^2+2mA)^2+4m^2B^2\}$,

$B_{25}=e^A (B_9CosB- B_{10}SinB)- e^{-A} (B_{15}CosB+B_{16}SinB)$,

$B_{26}=e^A (B_{10}CosB+B_9SinB)- e^{-A} (B_{16}CosB-B_{15}SinB)$,

$B_{27}=(AB_{25}+BB_{26})/\{2(A^2+B^2)\}$,

$B_{28}=(AB_{26}-BB_{25})/\{2(A^2+B^2)\}$,

$B_{29}=B_{15}SinhACosB-B_{16}CosASinhB$,

$B_{30}=B_{16}SinhACosB+B_{15}CosASinhB$,

$B_{31}=\{B_{29}(A^2-B^2)+2B_{30}AB\}/[2\{(A^2-B^2)^2+4A^2B^2\}]$,

$B_{32}=\{B_{30}(A^2-B^2)-2B_{29}AB\}/[2\{(A^2-B^2)^2+4A^2B^2\}]$,

$B_{33}=B_{17}B_{21}- B_{18}B_{22}$,

$B_{34}=B_{18}B_{21}+ B_{17}B_{22}$,

$B_{35}=B_{19}B_{23}-B_{20}B_{24}$,

$B_{36}=B_{20}B_{23}+B_{19}B_{24}$,

$B_{37}=\{(B_{33}+B_{35}+B_{27}+B_{31})\text{SinhACosB}+(B_{34}+B_{36}+B_{28}+B_{32})\text{CosASinhB}\}/\{2(\text{Sinh}^2\text{ACos}^2\text{B}+ \text{Cos}^2\text{A Sinh}^2\text{B})\}$,

$B_{38}=\{(B_{34}+B_{36}+B_{28}+B_{32})\text{SinhACosB}-(B_{33}+B_{35}+B_{27}+B_{31})\text{CosASinhB}\}/\{2(\text{Sinh}^2\text{ACos}^2\text{B}+ \text{Cos}^2\text{A Sinh}^2\text{B})\}$,

$B_{39}=B_{21}(B_5+B_{13})-B_{22}(B_6+B_{14})$,

$B_{40}=B_{22}(B_5+B_{13})+B_{21}(B_6+B_{14})$,

$B_{41}=B_{23}(B_7+B_{11})-B_{24}(B_8+B_{12})$,

$B_{42}=B_{24}(B_7+B_{11})+B_{23}(B_8+B_{12})$,

VI.4 RESULTS AND DISCUSSION

The variation of the temperature distribution with ζ at R = 7, P = 6, ζ = 5, and ϵ =0.02, k =15,m = 10, E = 5 for different values of τ_1= 1, 1.2, 3 when $\tau = \pi/3$ and $2\pi/3$ is shown in fig (1) and fig (2) respectively. From fig (1) and fig (2), the graph of the temperature variation is parabolic with vertex downwards. It is also clear from these figures that the temperature is minimum at the middle of the gap-length and remains negative throughout the gap-length except near the surface of the lower disc. It is seen from fig (1) that temperature increases with an increase in elastic-viscous parameter τ in the first half and being overlapped in the second half of the gap-length. It is observed from fig(2) that the temperature decreases with an increase in τ_1 in the middle of the gap-length and is being overlapped thereafter.

The variation of the temperature distribution with ζ at τ_1= 5, P = 6, ζ = 5, ϵ =0.02, k =15,m = 10, E = 5 for different values of R = 1, 1.5, 2 when $\tau = \pi/3$ and $2\pi/3$ is shown in fig (3) and fig (4) respectively. From fig (3) and fig (4), the graph of the temperature variation is parabolic with vertex downwards. It is also evident from these figures that the temperature is minimum at the middle of the gap length and remains negative throughout the gap-length except near the surface of the lower disc. It is also. It is also clear from these that temperature decreases with an increase in Reynolds number R throughout the gap-length.

The variation of the temperature distribution with ζ at R = 7, τ_1= 5, P = 6, ζ = 5, ϵ =0.02, k =15, E = 5 for different values of m= 1, 10, 20 when $\tau = \pi/3$ and $2\pi/3$ is shown in fig (5) and fig (6) respectively. From fig (5) and fig (6), the graph of the temperature variation is parabolic with vertex downwards. It is also observed from these figures that the temperature is minimum at the middle of the gap-length and remains negative throughout the gap-length except near the surface of the lower disc. It is also seen from fig (5) that the temperature increases with an increase in magnetic field parameter m and start overlapping near the upper disc and from fig (6), temperature decreases with an increase in magnetic field parameter m n the first half and being overlapped in the second half of the gap-length.

The variation of the temperature distribution with ζ at R = 7, τ_1= 5, P = 6, ζ = 5, ϵ =0.02, k =15, E = 5 for different values of Suciton parameter k = 1, 2, 3 when $\tau = \pi/3$ and $2\pi/3$ is shown in fig (7) and fig (8) respectively. It is observed from fig (7) and fig (8), that the temperature decreases, attains its minimum value at the middle of the gap length approximately and increases thereafter upto the surface of the upper disc for all values of suction parameter k. It is alos evident from these figure that temperature decreases with an increase in suction parameter k throughout the gap-length.

The variation of the nusselt number Nu_a with ξ at m = 10, R = 7, τ_1= 5, P = 6, ξ_0 = 5, $\in$ =0.02, E = 5, τ=π/3 for different values of Suciton parameter k = 1, 3, 7 is shown in fig (9). It is seen from this figure that the Nusselt number decreases with an increase in ξ throughout the gap-length. It is also observed from this figure that Nusselt number decreases with an increase in suction parameter k throughout the gap-length.

The variation of the Nusselt number Nu_b with ξ at m = 10, R = 7, τ_1= 5, P = 6, ξ_0 = 5, $\in$ =0.02, E = 5, τ= π/3 for different values of suction parameter k = 1, 3, 7 is shown in fig (10). It is clear from this figure that the Nusselt number increases with an increase in ξ throughout the gap-length. It is also evident from this figure that Nusselt number increases with an increase in suction parameter k throughout the gap-length.

	τ=π/3			τ=2π/3		
ζ	τ_1=1	τ_1=1.2	τ_1=3	τ_1=1	τ_1=1.2	τ_1=3
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	-65.224573	-63.508105	-50.823216	-55.365112	-57.795642	-60.353514
0.2	-104.48718	-102.76719	-93.291983	-104.60000	-106.38412	-103.98665
0.3	-131.17531	-129.91209	-125.65822	-136.99527	-137.92133	-133.58409
0.4	-148.36829	-147.58761	-146.37158	-154.22924	-154.56405	-150.97275
0.5	-154.97601	-154.57751	-154.39989	-158.69024	-158.70867	-156.58863
0.6	-149.56839	-149.41786	-149.35704	-151.32363	-151.22287	-150.24029
0.7	-131.41763	-131.39447	-131.24821	-132.08683	-131-97639	-131.61004
0.8	-100.37714	-100.39734	-100.22349	-100.63293	-100.55537	-100.45342
0.9	-56.540687	-56.55853	-56.43993	-56.659944	-56.622452	-56.618100
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table (1) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different elastic-viscous parameter τ_1	Table (2) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different elastic-viscous parameter τ_1

	τ=π/3			τ=2π/3		
ζ	R = 1	R=1.5	R = 2	R= 1	R =1.5	R = 2
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	-6.807930	-10.632880	-14.509118	-8.293030	-12.547987	-16.770010
0.2	-12.794078	-19.605990	-26.526349	-15.367564	-22.788340	-30.151436
0.3	-17.064457	-26.055892	-35.212122	-20.293759	-29.866305	-39.372527
0.4	-19.670820	-30.024408	-40.586111	-23.151638	-33.933500	-44.658184
0.5	-20.627837	-31.498287	-42.597465	-24.016546	-35.120653	-46.189183
0.6	-19.926768	-30.438433	-41.172678	-22.950446	-33.524558	-44.088002
0.7	-17.545350	-26.798943	-36.243541	-19.997716	-29.204494	-38.419903
0.8	-13.453976	-20.537097	-27.759201	-15.184341	-22.184447	-29.201264
0.9	-7061846	-11.615817	-15.686263	-8.519641	12.458753	-16.410514
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Table (3) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different Reynolds number R			Table (4) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different Reynolds number R		

	τ=π/3			τ=2π/3		
ζ	m = 1	m= 10	M = 20	$m = 1$	$m = 10$	$m = 20$
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	-55.537515	-53.838227	-48.970727	-56.014093	-58.214563	-63.727949
0.2	-99.627889	-97.076999	-89.842277	-100.24898	-102.64438	-107.89217
0.3	-131.21446	-128.71129	-121.88663	-131.83270	-133.60911	-136.52111

0.4	-150.23720	-148.25181	-143.20599	-150.82293	-151.89202	-152.63983
0.5	-156.66307	-155.29437	-152.18139	-157.23380	-157.77939	-157.27195
0.6	-150.48558	-149.64148	-148.02760	-151.06036	-151.29009	-150.34945
0.7	-131.71717	-131.24971	-130.57927	-132.28754	-132.35229	-131.46825
0.8	-100.37986	-100.15209	-99.960704	-100.88642	-100.88179	-100.26189
0.9	-56.486635	-56.398469	-56.372249	-56.808617	-56.79387	-56.49193
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Table (5) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different magnetic field parameter m			Table (6) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different magnetic field parameter m		

	$\tau=\pi/3$			$\tau=2\pi/3$		
ζ	k = 1	k = 2	k = 3	k = 1	k = 2	k = 3
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	-1.015145	-4.788223	-8.561300	-5.109762	-8.902962	-12.696162
0.2	-3.184775	-9.891362	-16.597950	-8.304388	-15.042959	-21.781530
0.3	-5.469714	-14.272684	-23.075654	-9.856384	-18.65864	-27.535345
0.4	-7.389866	-17.451434	-27.513001	-10.506919	-20.605855	-30.704791
0.5	-8.556801	-19.038056	-29.519311	-10.519122	-21.037713	-31.556304
0.6	-8.779536	-18.841103	-28.902760	-9.904991	-20.003926	-30.102862
0.7	-8.008123	-16.811093	-25.614063	-8.599570	-17.439050	-26.278531
0.8	-6.259874	-12.966461	-19.673048	-6.541793	-13.280365	-20.018936
0.9	-3.575387	-7.348464	-11.121541	-3.689072	-7.482272	-11.275472
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Table (7) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different suction parameter m			Table (8) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different suction parameter m		

	Nu_a			Nu_b		
ζ	k = 1	k = 3	k = 7	k = 1	k = 3	k = 7
0	-10.061270	-11.835657	-15.384432	2.371043	4.145431	7.694205
1	-10.469546	-12.243933	-15.792708	2.387274	4.161661	7.710436

2	-11.694374	-13.468761	-17.017536	2.435965	4.210352	70759127
3	-13.735753	-15.510141	-19.058915	2.517117	4.291504	7.840279
4	-16.593685	-18.368072	-21.916847	2.630729	4.405116	7.953891
5	-20.268169	-22.042556	-25.591331	2.776802	4.551190	8.099964
6	-24.759205	-26.533592	-30.082366	2.955336	4.729724	8.278498
7	-30.066792	-31.841179	-35.389954	3.166331	4.940718	8.489493
8	-36.190932	-37.965319	-41.514094	3.409787	5.184174	8.732949
9	-43.131623	-44.906010	-48.454785	3.686703	5.460090	9.008865
10	-50.888866	-52.663254	-56.212028	3.994080	5.768467	9.317242
	Table (9) variation of Nusselt number Nu_a at different suction parameter k			Table (10) variation of Nusselt number Nu_a at different suction parameter k		

	$\tau=\pi/3$			$\tau=2\pi/3$		
ζ	$\tau_1=1$	$\tau_1=1.2$	$\tau_1=3$	$\tau_1=1$	$\tau_1=1.2$	$\tau_1=3$
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	-65.224573	-63.508105	-50.823216	-55.365112	-57.795642	-60.353514
0.2	-104.48718	-102.76719	-93.291983	-104.60000	-106.38412	-103.98665
0.3	-131.17531	-129.91209	-125.65822	-136.99527	-137.92133	-133.58409
0.4	-148.36829	-147.58761	-146.37158	-154.22924	-154.56405	-150.97275
0.5	-154.97601	-154.57751	-154.39989	-158.69024	-158.70867	-156.58863
0.6	-149.56839	-149.41786	-149.35704	-151.32363	-151.22287	-150.24029
0.7	-131.41763	-131.39447	-131.24821	-132.08683	-131-97639	-131.61004
0.8	-100.37714	-100.39734	-100.22349	-100.63293	-100.55537	-100.45342
0.9	-56.540687	-56.55853	-56.43993	-56.659944	-56.622452	-56.618100
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Table (1) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different elastic-viscous parameter τ_1			Table (2) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different elastic-viscous parameter τ_1		

	$\tau=\pi/3$			$\tau=2\pi/3$		
ζ	R = 1	R=1.5	R = 2	***R***= 1	***R*** =1.5	***R*** = 2

0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	-6.807930	-10.632880	-14.509118	-8.293030	-12.547987	-16.770010
0.2	-12.794078	-19.605990	-26.526349	-15.367564	-22.788340	-30.151436
0.3	-17.064457	-26.055892	-35.212122	-20.293759	-29.866305	-39.372527
0.4	-19.670820	-30.024408	-40.586111	-23.151638	-33.933500	-44.658184
0.5	-20.627837	-31.498287	-42.597465	-24.016546	-35.120653	-46.189183
0.6	-19.926768	-30.438433	-41.172678	-22.950446	-33.524558	-44.088002
0.7	-17.545350	-26.798943	-36.243541	-19.997716	-29.204494	-38.419903
0.8	-13.453976	-20.537097	-27.759201	-15.184341	-22.184447	-29.201264
0.9	-7061846	-11.615817	-15.686263	-8.519641	12.458753	-16.410514
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Table (3) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different Reynolds number R			Table (4) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different Reynolds number R		

	τ=π/3			τ=2π/3		
ζ	m = 1	m= 10	M = 20	$m = 1$	$m = 10$	$m = 20$
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	-55.537515	-53.838227	-48.970727	-56.014093	-58.214563	-63.727949
0.2	-99.627889	-97.076999	-89.842277	-100.24898	-102.64438	-107.89217
0.3	-131.21446	-128.71129	-121.88663	-131.83270	-133.60911	-136.52111
0.4	-150.23720	-148.25181	-143.20599	-150.82293	-151.89202	-152.63983
0.5	-156.66307	-155.29437	-152.18139	-157.23380	-157.77939	-157.27195
0.6	-150.48558	-149.64148	-148.02760	-151.06036	-151.29009	-150.34945
0.7	-131.71717	-131.24971	-130.57927	-132.28754	-132.35229	-131.46825
0.8	-100.37986	-100.15209	-99.960704	-100.88642	-100.88179	-100.26189
0.9	-56.486635	-56.398469	-56.372249	-56.808617	-56.79387	-56.49193
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Table (5) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different magnetic field parameter m			Table (6) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different magnetic field parameter m		

	τ=π/3	τ=2π/3

ζ	k = 1	k = 2	k = 3	k = 1	k = 2	k = 3
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	-1.015145	-4.788223	-8.561300	-5.109762	-8.902962	-12.696162
0.2	-3.184775	-9.891362	-16.597950	-8.304388	-15.042959	-21.781530
0.3	-5.469714	-14.272684	-23.075654	-9.856384	-18.65864	-27.535345
0.4	-7.389866	-17.451434	-27.513001	-10.506919	-20.605855	-30.704791
0.5	-8.556801	-19.038056	-29.519311	-10.519122	-21.037713	-31.556304
0.6	-8.779536	-18.841103	-28.902760	-9.904991	-20.003926	-30.102862
0.7	-8.008123	-16.811093	-25.614063	-8.599570	-17.439050	-26.278531
0.8	-6.259874	-12.966461	-19.673048	-6.541793	-13.280365	-20.018936
0.9	-3.575387	-7.348464	-11.121541	-3.689072	-7.482272	-11.275472
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Table (7) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different suction parameter m			Table (8) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different suction parameter m		

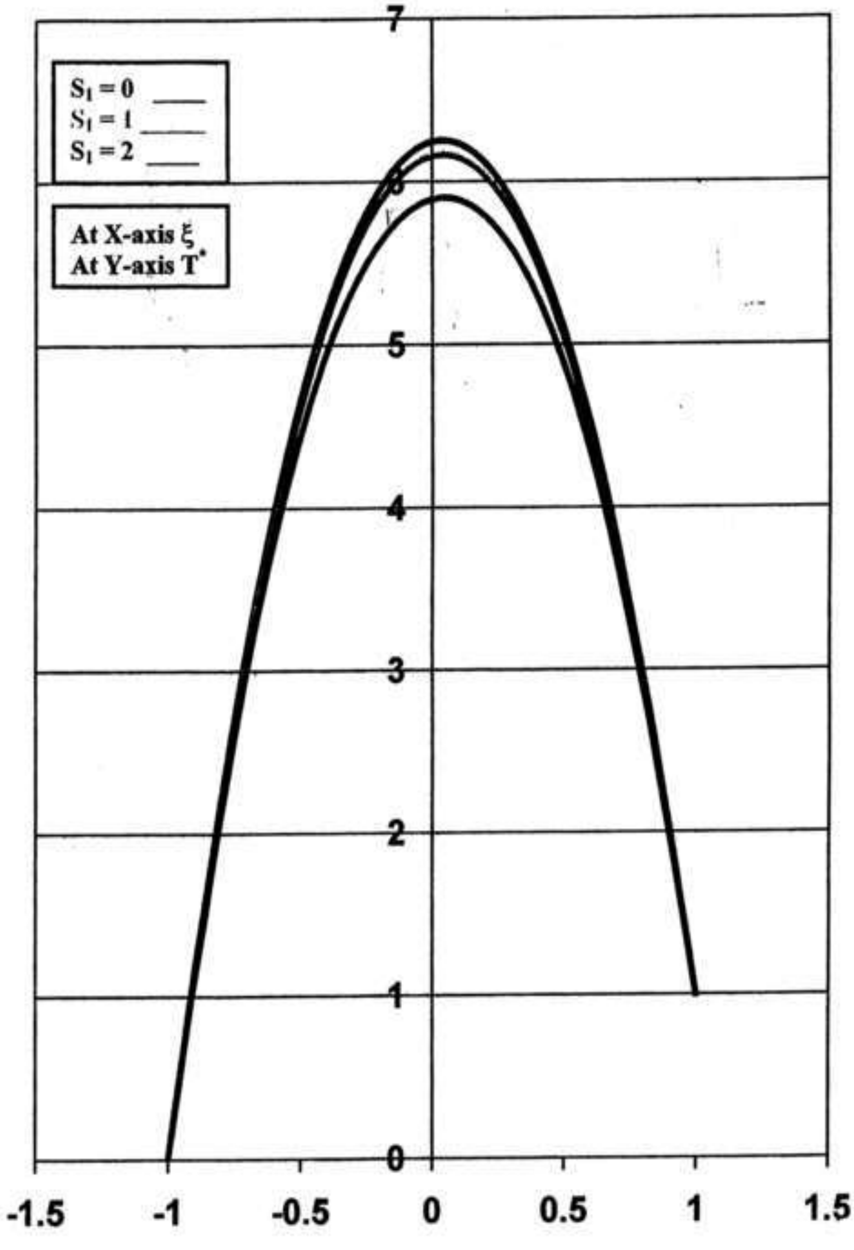
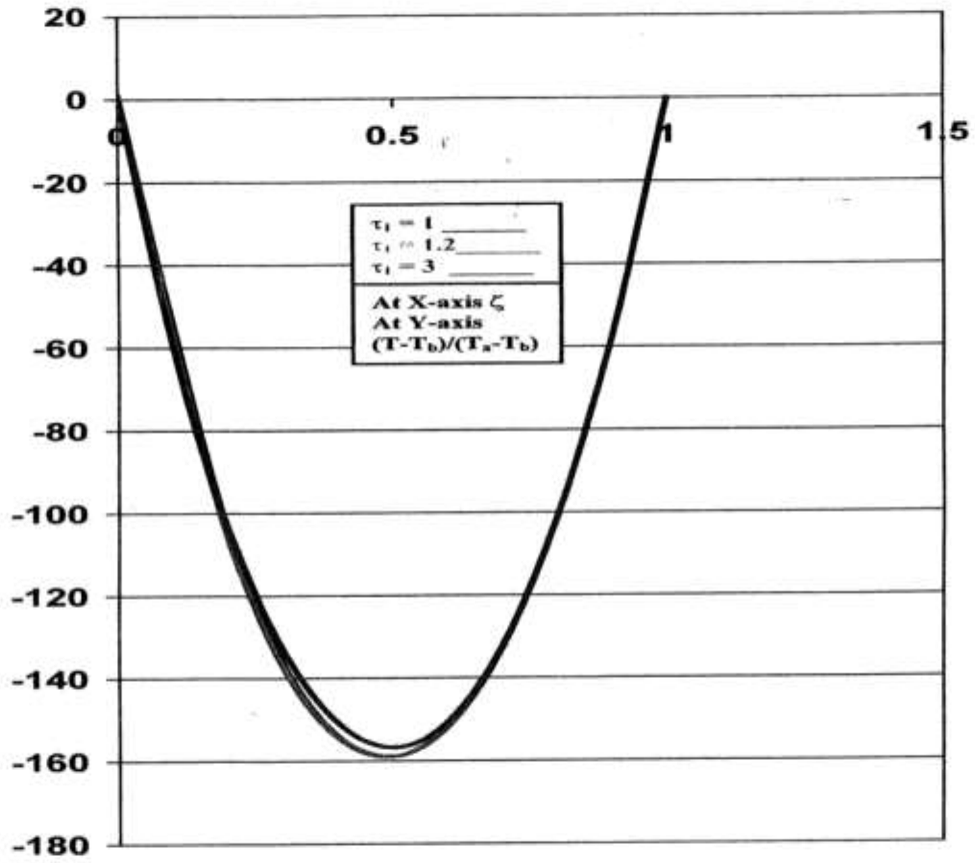
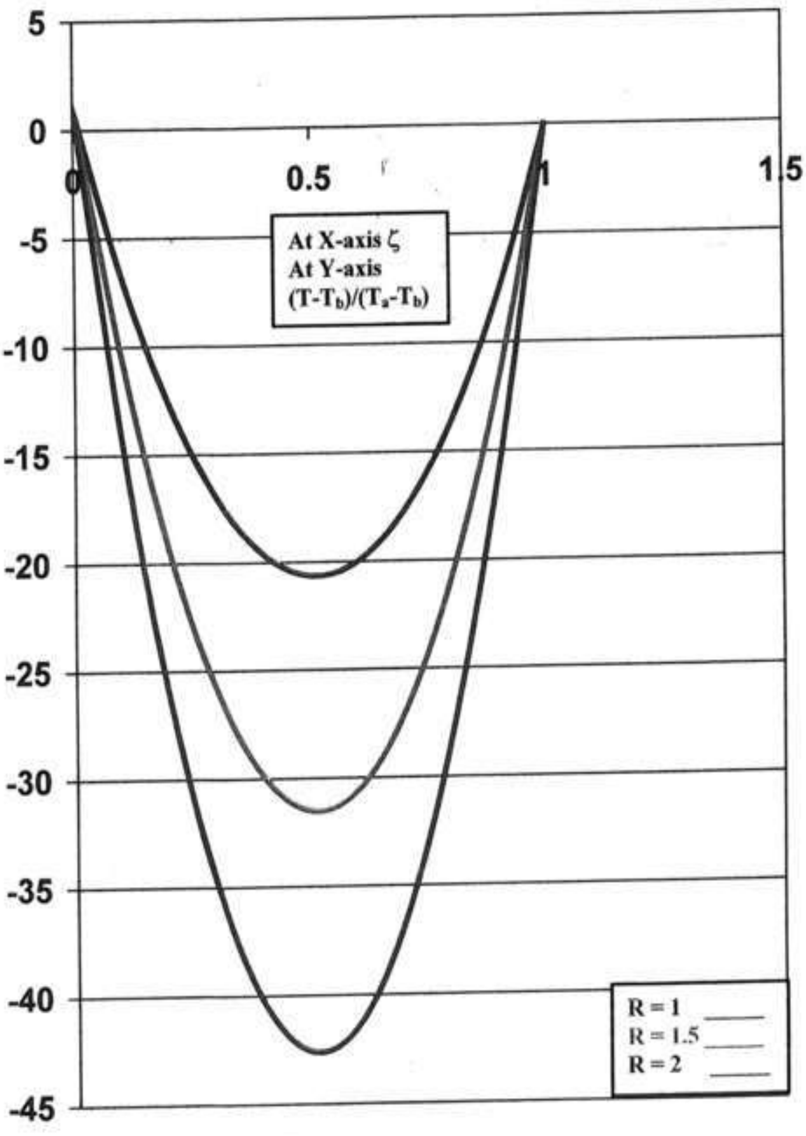


Fig (3) Variation of the temperature T* With ξ for different values of Hartman Number S_1.

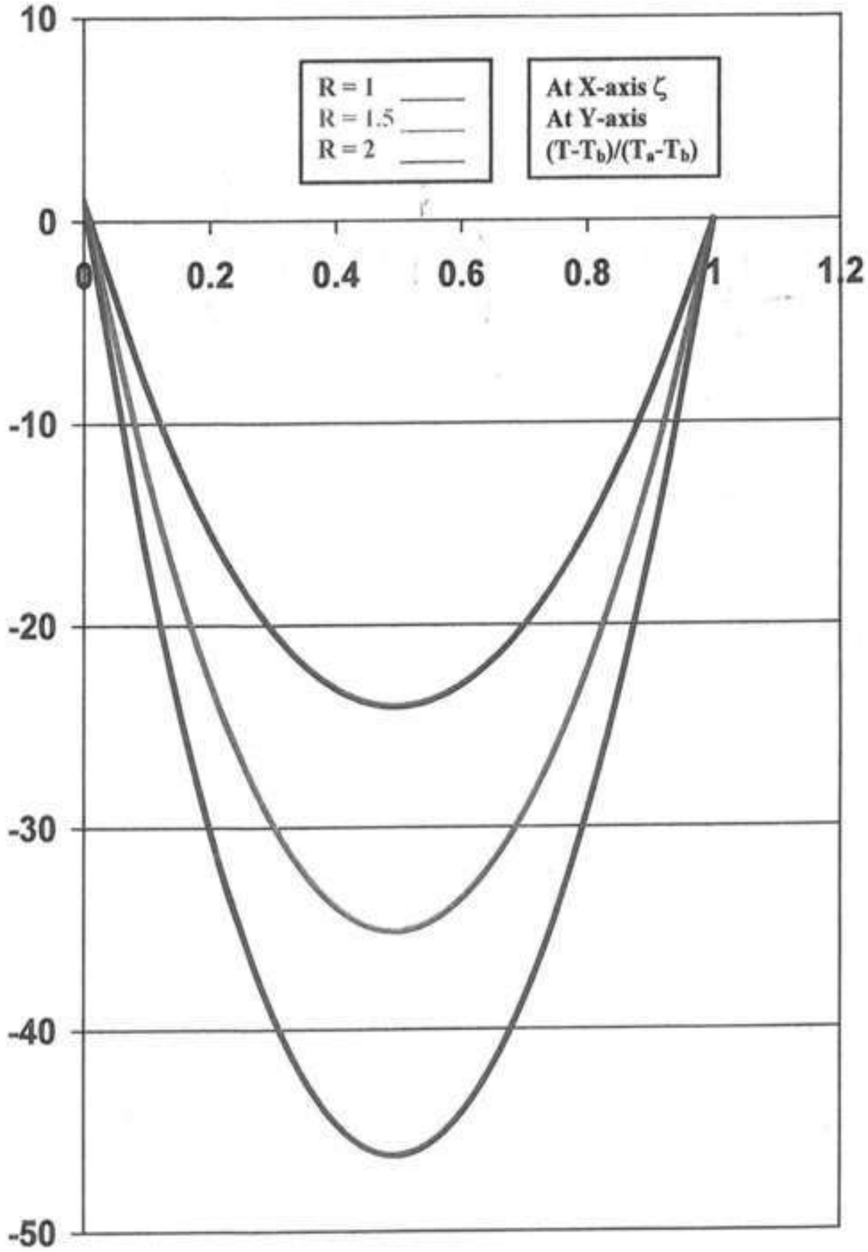
	Nu_a			Nu_b		
ζ	k = 1	k = 3	k = 7	k = 1	k = 3	k = 7
0	-10.061270	-11.835657	-15.384432	2.371043	4.145431	7.694205
1	-10.469546	-12.243933	-15.792708	2.387274	4.161661	7.710436
2	-11.694374	-13.468761	-17.017536	2.435965	4.210352	70759127
3	-13.735753	-15.510141	-19.058915	2.517117	4.291504	7.840279
4	-16.593685	-18.368072	-21.916847	2.630729	4.405116	7.953891
5	-20.268169	-22.042556	-25.591331	2.776802	4.551190	8.099964
6	-24.759205	-26.533592	-30.082366	2.955336	4.729724	8.278498
7	-30.066792	-31.841179	-35.389954	3.166331	4.940718	8.489493
8	-36.190932	-37.965319	-41.514094	3.409787	5.184174	8.732949
9	-43.131623	-44.906010	-48.454785	3.686703	5.460090	9.008865
10	-50.888866	-52.663254	-56.212028	3.994080	5.768467	9.317242
	Table (9) variation of Nusselt number Nu_a at different suction parameter k			Table (10) variation of Nusselt number Nu_a at different suction parameter k		



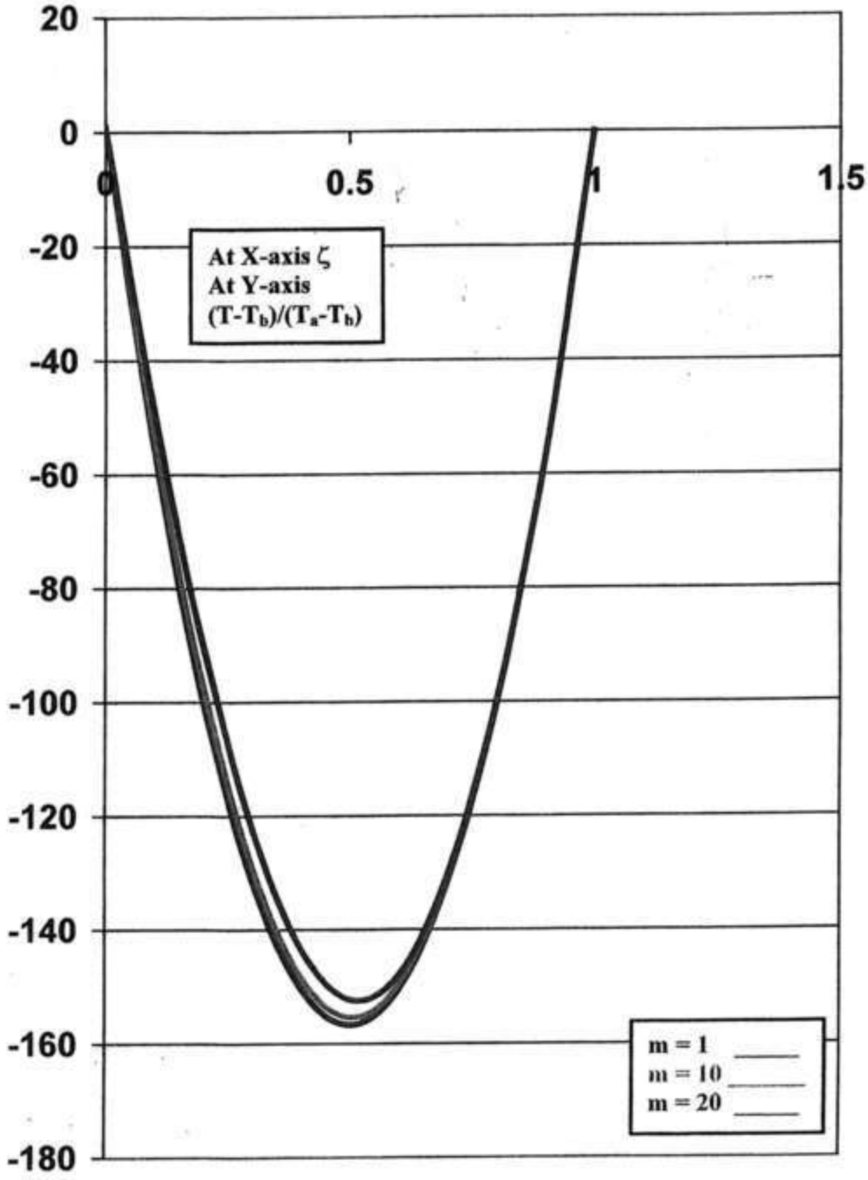
Fig(2) variation of temperature distribution $(T-T_b)/(T_s-T_b)$ at different elastico-viscous parameter τ_1 at $\tau = 2\pi/3$.



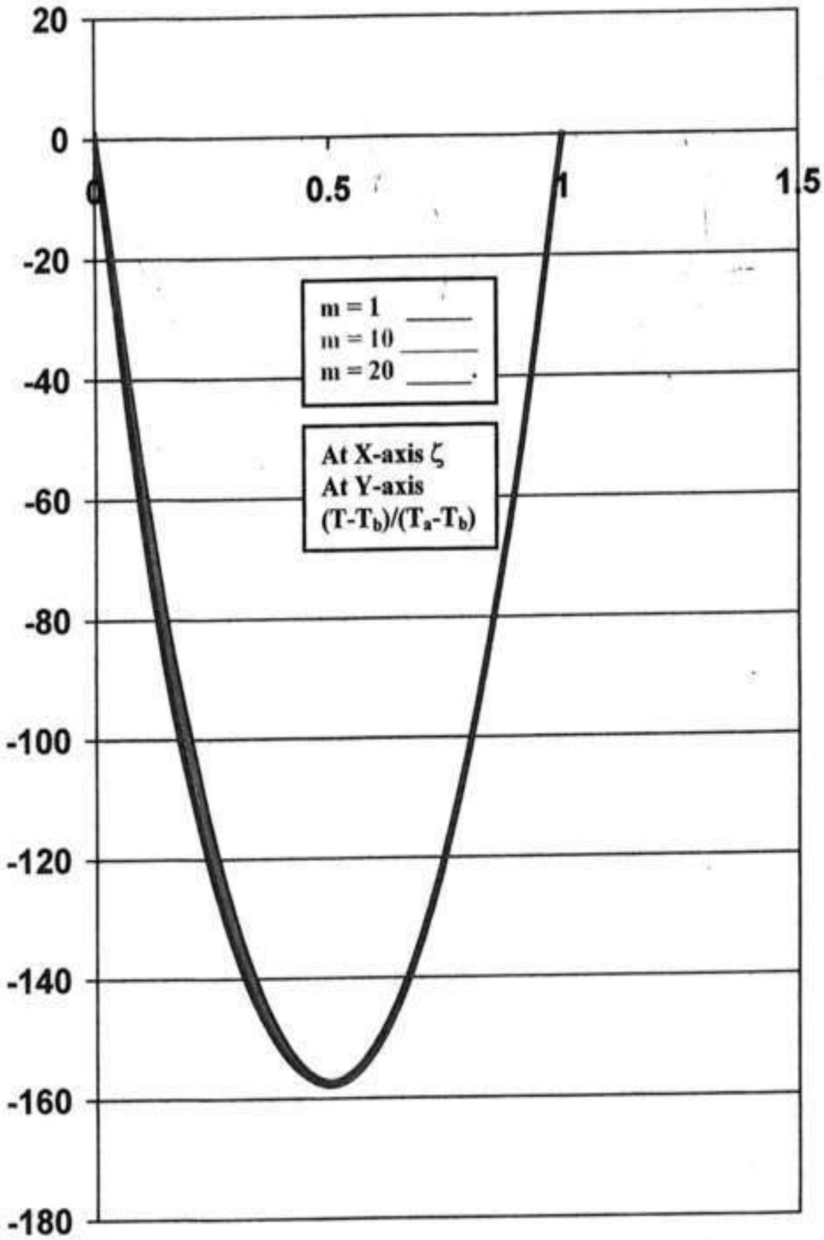
Fig(3) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different Reynolds number R at $\tau = \pi/3$.



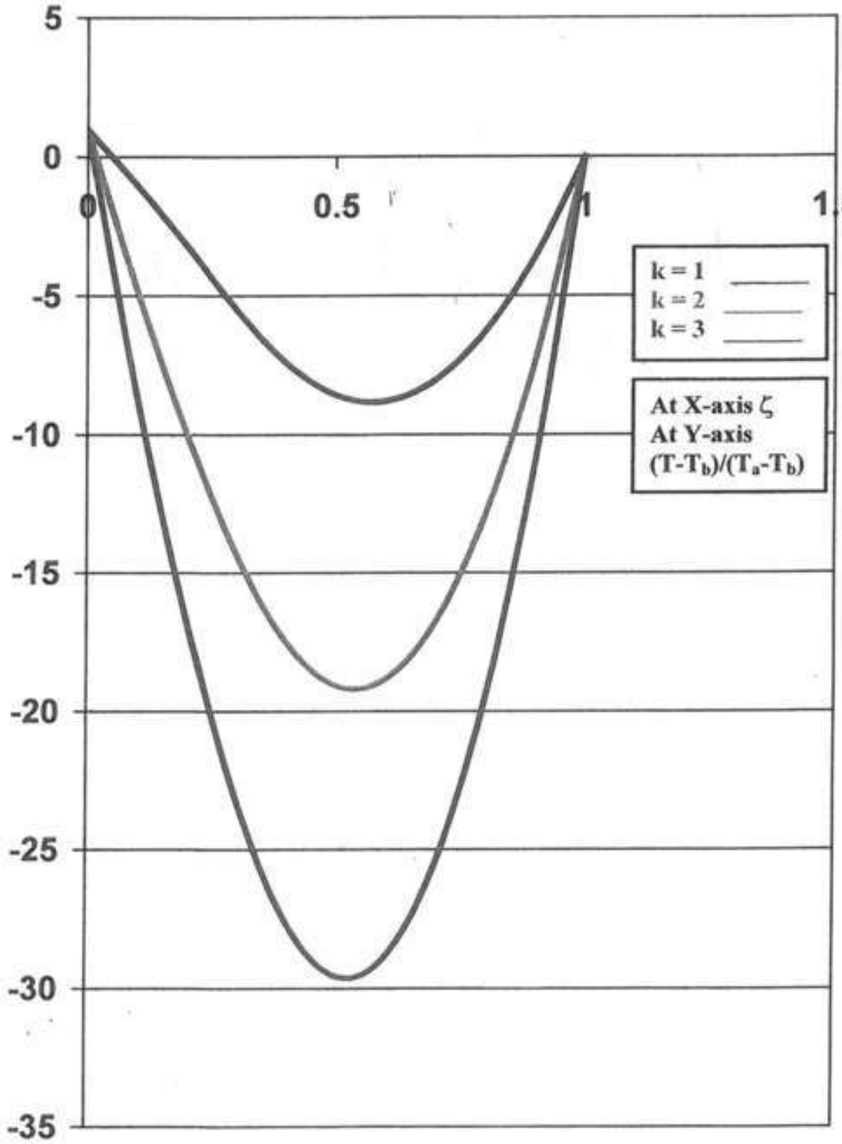
Fig(4) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different Reynolds number R at $\tau = 2\pi/3$.



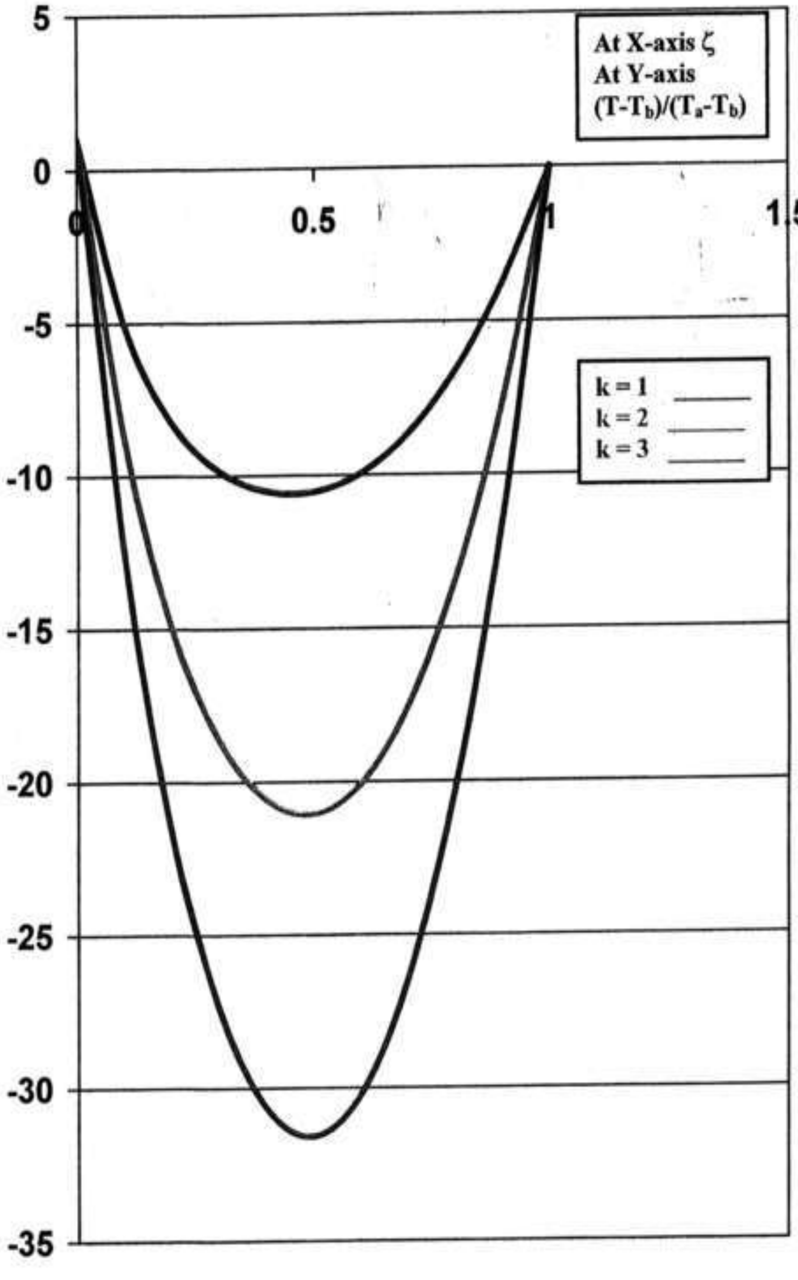
Fig(5) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different magnetic field m at $\tau = \pi/3$.



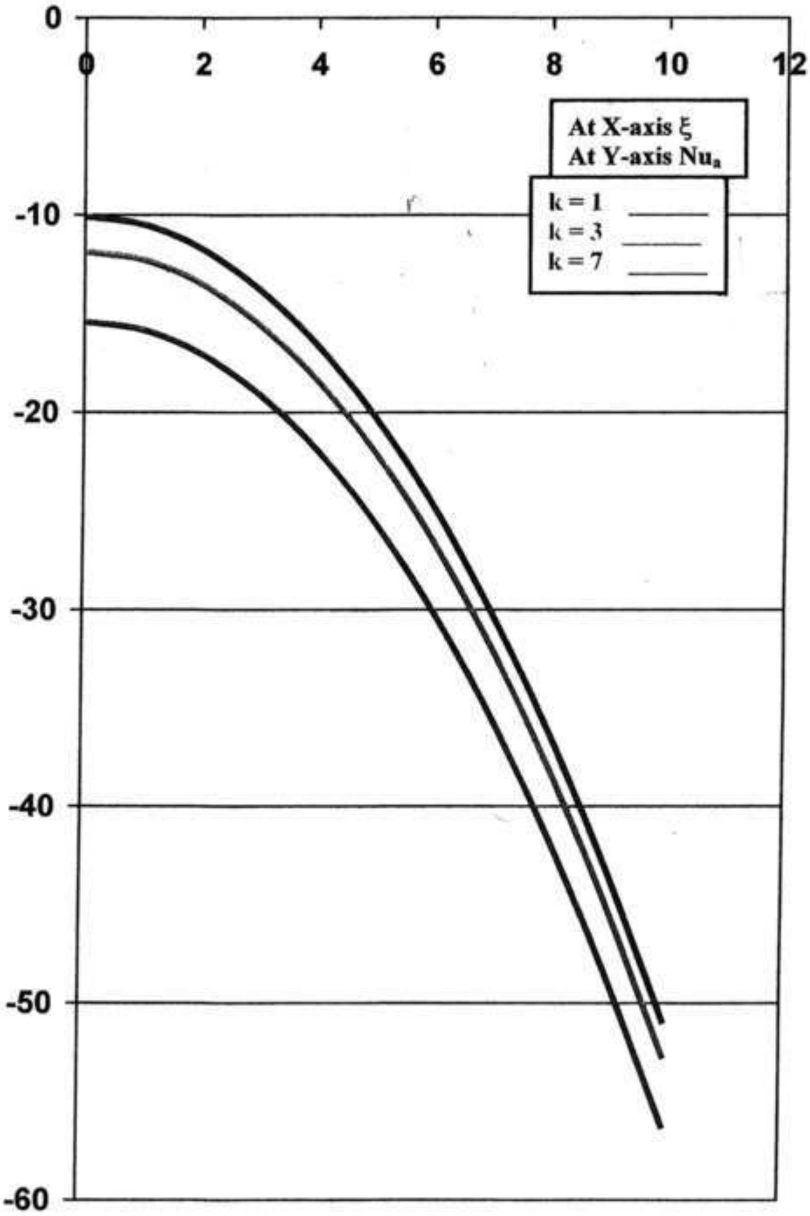
Fig(6) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different magnetic field m at $\tau = 2\pi/3$.



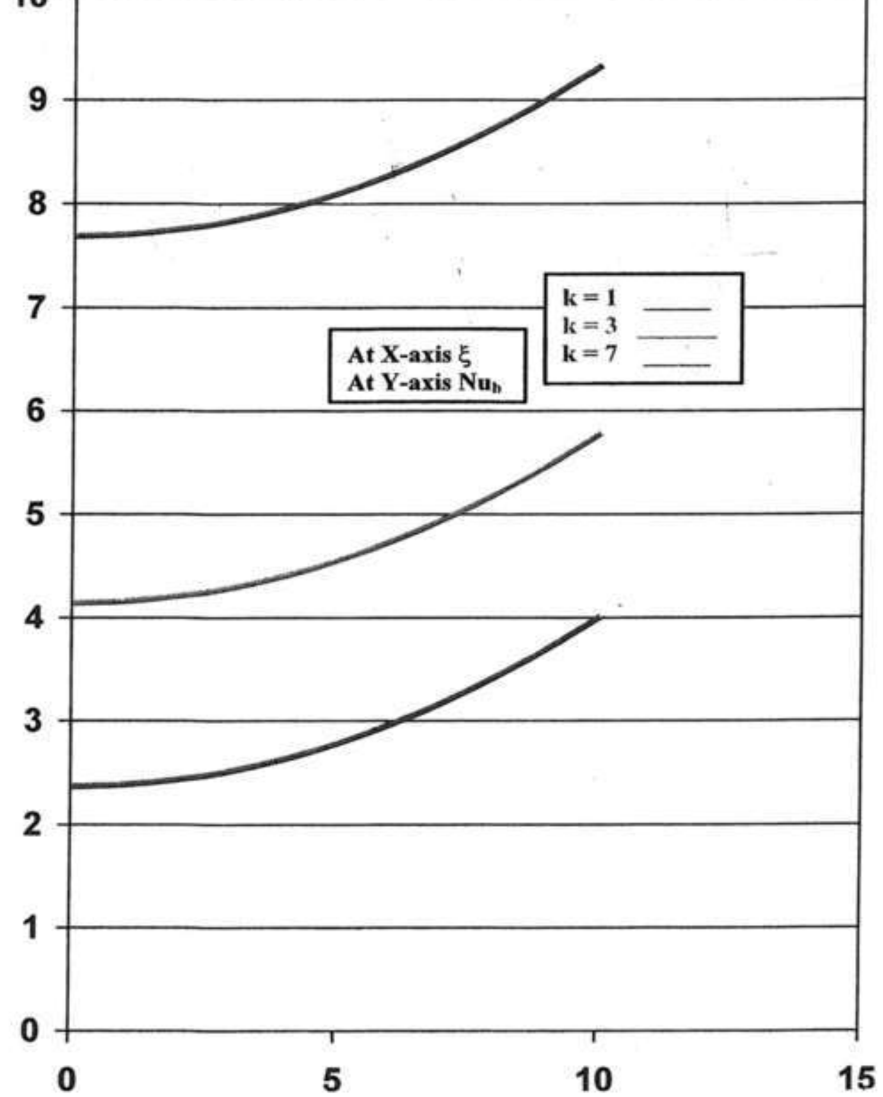
Fig(7) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different suction parameter k at $\tau = \pi/3$.



Fig(8) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different suction parameter k at $\tau = 2\pi/3$.



Fig(9) variation of Nusselt number Nu_a at different suction parameter k at $\tau = \pi/3$.



Fig(10) variation of Nusselt number Nu_b at different suction parameter k at $\tau = \pi/3$.

Chapter No. 7

Heat transfer in the flow of a non-Newtonian second –order fluid 171 second-order fluid between two enclosed torsion ally oscillating discs with uniform suction and injection in the presence of the magnetic field.

VII.1 INTRODUCTION

The non-Newtonian effects are exhibited through two dimensionless parameters $\tau_1(=n\mu_2/\mu_1)$ and $\tau_2(=n\mu_3/\mu_1)$, where μ_1, μ_2, μ_3 are coefficient of Newtonian viscosity, elastic-viscosity and cross viscosity. n being the uniform frequency of the oscillation. The variation of temperature distribution with elastic-viscous parameter τ_1, cross-viscous parameter τ_2 (based on the relation $\tau_1 = \alpha\tau_2$, where $\alpha = -0.2$ as for 5.46% poly-iso- butylenes type solution in cetane at 30^0 C (Markowitz[38]) Reynolds number R, magnetic field m, suction parameter k at different phases difference τ is shown graphically.

VII.2 FORMULATION OF THE PROBLEM

In the three dimensional cylindrical set of co-ordinates (r, θ, z) the system consists of a finite oscillating disc of radius r_s (coinciding with the plane z = 0) performing rotator oscillations of the type rΩCost of small amplitude ϵ , about perpendicular axis r = 0 with a constant angular velocity Ω in an incompressible second-order fluid forming the part of a cylindrical casing or housing. . The symmetrical radial steady outflow has a small mass rate 'm' of radial outflow ('**-m**' for net radial inflow). The inlet condition is taken as a simple radial source flow along z-axis starting from radius r_0. A constant magnetic field B_0 is applied normal to the plane of the oscillating disc. The induced magnetic field is neglected. The lower disc z = 0 is maintained at constant temperature T_a while the upper disc $z = z_0$ at constant temperature T_b.

Assuming (u, v, w) as the velocity components along the cylindrical system of axes (r, θ, z) the relevant boundary conditions of the problem are:

$z = 0,\qquad u = 0,\qquad v = r\Omega^{i\tau}(\text{Real part}),\qquad w = w_0\qquad T = T_a$

$z = z_0,\qquad u = 0,\qquad v = r\Omega^{i\tau}(\text{Real part}),\qquad w = w_0\qquad T = T_b$

(7.1)

where the gap z_0 is assumed small in comparison with the disc radius r_s. The velocity components for the axisymmetric flow compatible with the continuity criterion can be taken as [64,65,66],

$U = -\,\xi H'(\zeta,\tau)+(R_m/R_z)\, M'(\zeta,\tau)/\xi,$

$V = \xi G(\zeta,\tau)+(R_L/R_z)\, L(\zeta,\tau)/\xi,$

$W = 2H(\zeta,\tau).$

(7.2)

and for the temperature, we take

$T = T_b+(v_1\Omega/C_v)\{\phi(\zeta,\tau)+\xi^2\Psi(\zeta,\tau)\}$

(7.3)

where $U = u/\Omega z_0$, $V= v/ \Omega z_0$, $W = w/\Omega z_0$, $\xi= r/z_0$, ζ,τ are dimensionless quantities and $H(\zeta,\tau)$, $G(\zeta,\tau)$, $L(\zeta,\tau)$, $M'(\zeta,\tau)$, $\phi(\zeta,\tau)$, are dimensionless function of the dimensionless variables $\zeta=z/z_0$ and $\tau = nt$. $R_m(=m/2\pi p z_0 v_1)$, R_L $(=\mathbf{L}/2\pi p z_0 v_1)$ are dimensionless number to be called the Reynolds number of net outflow and circulatory flow respectively. R_z $(=\Omega z_0^2/v_1)$ be the flow Reynolds number. The small mass rate 'm' of the radial outflow is represented by

$$\mathbf{m} = 2\pi p\int_0^{z_0} r\,u\,dz$$

(7.4)

Using expression (7.2) and (7.3), the boundary condition (7.1) transform for G, L & H into the following form:

$G(0, \tau) = \text{Real}(e^{i\tau})$, $G(1, \tau) = \text{Real}(e^{i\tau})$,

$L(0, \tau) = 0$, $L(1, \tau) = 0$,

$H(0, \tau) = k$, $H(1, \tau) = 0$,

$H'(0, \tau) = 0$, $H'(1, \tau) = 0$,

$\phi(0, \tau) = 1/E = S$, $\phi(1, \tau) = 0$,

$\Psi(0, \tau) = 0$, $\Psi(1, \tau) = 0$

(7.5)

where E $[=\Omega v_1/\{C_v(T_a-T_b)\}]$ is the Eckert number and k $[=w_0/2\Omega z_0]$ is the suction

parameter.

The conditions on M on the boundaries are obtainable form the expression

(7.4) for **m** as follows:

$M(1, \tau) - M(0, \tau) = 1$

(7.6)

which on choosing the discs as streamlines reduces to

$M(1, \tau) = 1$, $M(0, \tau) = 0$

(7.7)

Using eqs. (1.4) and expression (7.2) in equation (1.8) and neglecting the squares

& higher powers of R_m/R_z (assumed small), we have the following equations in dimensionless form:

$-(1/pz_0)(\partial p/\partial \xi)$=-n$\Omega$z$_0$\{$\xi\partial$H'-(R$_m$/R$_z$)($\partial M'/\xi$)\}+$\Omega^2z_0\xi$(H'^{2}-2HH''-G^2)+$\Omega^2z_0$ (R$_m$/R$_z$)(2HM''ξ)-Ω^2z$_0$(R$_L$/R$_z$)(2LG/ξ)+(v$_1\Omega$/z$_0$)\{H'''ξ-(R$_m$/R$_z$)(M'''/ξ)\}- (2v$_2$/z$_0$) [nΩ/2] \{(R$_m$/R$_z$) ($\partial M'''/\xi$)- $\xi\partial H$'''\} + Ω^2 ξ(H''^{2}-HH^{iv}) + (R$_m$/R$_z$) (Ω^2/ξ) (H'''M'+H''M''+H'M'''+HM^{iv})-(R$_L$/R$_z$)(2Ω^2/ξ)(L'G'+LG'')]-(4v$_3\Omega^2$-z$_0$)\{(R$_m$/R$_z$) (1/2ξ) (H'''M'+H'M'''+H''M'')-(R$_L$/R$_z$)(1/2ξ)(2L'G'+LG'')+ (ξ/4) (H''^{2}-G'^{2}-2H'H''')\} + ($\sigma B_0{}^2\Omega$z$_0$/p)\{-ξH'+(R$_m$/R$_z$)(M'/ξ)\}.

(7.8)

0 = -nΩz$_0$\{$\xi\partial$G+(R$_L$/R$_z$)($\partial L/\xi$)\}-(2Ω^2z$_0\xi$)(HG'-H'G)- Ω^2z$_0$(R$_m$/R$_z$) (2M'G/ξ)-Ω^2z$_0$(R$_L$/R$_z$)(2HL'/ξ)+(v$_1\Omega$/z$_0$)\{ξG''+(R$_L$/R$_z$)(L''/ξ)\}+(2v$_2$/z$_0$)[(nΩ/2) \{$\xi\partial$G''+(R$_L$/R$_z$)($\partial L''/\xi$)\}+(R$_L$/R$_z$)(Ω^2/ξ)(H''L'+H'''L+HL'''+H'L'') +(Ω^2/ξ) (HG'''-H''G')+(R$_m$/R$_z$)(2Ω^2/ξ)(M'G''+M''G')]+(2v$_3\Omega^2$/z$_0$)\{ξ(H'G''-H''G') +(R$_L$+R$_z$)(1/ξ)(H''L'+H'''L+H'L'')+(R$_m$+R$_z$)(1/ξ)(2M''G'+M'G'')-($\sigma B_0{}^2\Omega$z$_0$/p) \{ξG+(R$_L$/R$_z$)(L/ξ)\}.

(7.9)

$-(1/pz_0)(\partial p/\partial \zeta)$=2n$\Omega$z$_0\partial$H+4$\Omega^2z_0$HH'-2v$_1\Omega$H''/z$_0$-(2v$_2$/z$_0$)\{n$\Omega\partial$H''+2$\Omega^2\xi^2$ (H''H'''+G'G'')+Ω^2(22H'HH''+2HH''')-(R$_m$/R$_z$)2Ω^2(H''M'''+H'''M'')+ (R$_L$/R$_z$) 2Ω^2 (L'G'' + L''G')\}-(2v$_3$ Ω^2/z$_0$)\{ξ^2(H''H''' +G'G'')+14H'H''-(R$_m$/R$_z$) (H''M'''+H'''M'')+(R$_L$/R$_z$)(L'G''+L''G')\}

(7.10)

$$pC_v(\partial T/\partial t+u\partial T/\partial r+w\partial T/\partial z) = K\{\partial^2T/\partial r^2+(1/r)\,\partial T/\partial r+\partial^2T/\partial z^2\}+\Phi$$

(7.11)

where

$$\Phi = \tilde{\tau}^i_j d^i_j$$

(7.12)

C_v is the specific heat at constant volume, Φ be the viscous-dissipation function, $\tilde{\tau}^i_j$ is the mixed deviatoric stress tensor, K is the thermal conductivity, p is the density of the fluid; B and σ are intensity of the magnetic field and conductivity of the fluid considered.

Differentiating (7.8) w.r.t ζ and (7.10) w.r.t ξ and then eliminating $\partial^2 p/\partial\zeta.\partial\xi$ from the equation thus obtained. We get

$$-n\Omega z_0\{\xi\partial H''-(R_m/R_z)\partial M''/\xi\}-2\Omega^2 z_0\xi(HH'''-GG')+(R_m/R_z)(2\Omega^2 z_0/\xi)(H'M''+HM''')-(R_L/R_z)(2\Omega^2 z_0/\xi)(LG'+L'G)-(v_1\Omega/z_0)\{(R_m/R_z)(M^{iv}/\xi)-\xi H^{iv}\}-(2v_2/z_0)[(n\Omega/2)\{(R_m/R_z)(\partial M^{iv}/\xi)-\xi\partial H^{iv}\}-\Omega^2\xi(2H''H'''+H'H^{iv}+HH^{v}+4G'G'')+(R_m/R_z)(\Omega^2/\xi)(2H'''M''+H^{iv}M'+2H''M'''+2H'M^{iv}+HM^{v})-(R_L/R_z)(2\Omega^2/\xi)(2L'G''+L''G'+LG''')]-(2v_3\Omega^2/z_0)\{(R_m+R_z)(1/\xi)(H^{iv}M'+2H'''M''+2H''M'''+H'M^{iv})-(R_L+R_z)(1/\xi)(3L'G''+2L''G'+LG''')-\xi(H'H^{iv}+3G'G''+2H''H''')\}+(\sigma B_0^2\Omega z_0/p)\{-\xi H+(R_m/R_z)(M''/\xi)\}=0$$

(7.13)

On equating the coefficients of ξ and $1/\xi$ from the equation (7.9) & (7.13), we get the following equations:

$$G''=R\partial G+2\in R(HG'-H'G)-\tau_1\partial G''-2\in\tau_1(HG'''-H''G')-2\in\tau_2(H'G''H''G')+m^2G$$

(7.14)

$$L'' = R\partial L + 2\in R(M'G+HL) - \tau_1\partial L'' - 2\in\tau_1(H''L'+H'''L+HL'''+H'L''+2M'G''+2M''G') - 2\in\tau_2(H''L'+H'''L+H'L''+2M''G'+M'G') + m^2L$$
(7.15)

$$H^{iv} = R\partial H + 2\in R(HH'''+GG') - \tau_1\partial H^{iv} - 2\in\tau_1(H'H^{iv}+HH^{v}+2H''H'''+4G'G'') - 2\in\tau_2(H'H^{iv}+2H''H''+3G'G'') + m^2H''$$
(7.16)

$$M^{iv} = R\partial M + 2\in R(H'M''+HM'''-LG'-L'G) - \tau_1\partial M^{iv} - 2\in\tau_1(2H'''M''+H^{iv}M'+2H''M'''+2H'M^{iv}+HM^{v}-4L'G''-2L''G'-2LG''') - 2\in\tau_2(H^{iv}M'+2H'''M''+2H''M'''+H'M^{iv}-3L'G''-2L''G'-LG''') + m^2M''$$
(7.17)

where $R(=nz_0/v_1)$ is the Reynolds number, $\tau_1(=nv_2/v_1)$, $\tau_2(=nv_3/v_1)$ and $\in(=\Omega/n)$ are the dimensionless parameter, $m^2 = \sigma B_0^2 z_0^2/\mu_1$ is the dimensionless magnetic field and $R_m/R_L =$ **m/L** ≈ 1.

Using (7.3), (7.12) in (7.11) and equating the coefficient of ξ^2 and independent term of, we get

$$\Psi'' = \in RP_r[\partial\Psi/\in - 2H'\Psi + 2H\Psi' - H''^2 + G'^2 - \tau_1(H''\partial H'' + G'\partial G') - 2\in\tau_1(H'H''^2 + H'G'^2HH''H''' + HG'G'') - 3\in\tau_2(H'H''^2 + H'G'^2)],$$
(7.18)

$$4\Psi + \phi'' = \in RP_r[\partial\phi/\in + (R_m/R_z)2M'\Psi + 2H\phi' - 12H'^2 + (R_m/R_z)2H''M'' - (R_L/R_z)2L'G' - \tau_1\{12H'\partial H' - (R_m/R_z)(H''\partial M'' + M''\partial H'') + (R_L/R_z)(G'\partial L' + L'\partial L')\} - 2\in\tau_1\{(12H'^3 + 12HH'H'' + (R_m/R_z)(2M'G'^2 - HH'''M'' - H''^2M' -$$

$HH''M''')+(R_L/R_z)(HL'G'+3H'L'G'+3LG'H''+HG'L'')\}-\in$
$\tau_2\{(R_m/R_z)(3M'G'^2-6H'H''M)+24H'^3+6(R_L/R_z)(H''LG'+H'L'G')\}]$
(7.19)

Where $P_r=\mu_1 C_v/K$ is the Prandtl number.

For $R_m=R_L=B_0=0$, the differential equations (7.8)-(7.10) are identical to those obtained by Sharma & Gupta[67)] (for $S_1= 1$, $S_2=0$), Sharma & Singh[68)] (for $S_1 =1$, $S_2=1$) and for $R_m=R_L=B_0=0$,the differential equations (7.8)-(7.10),(7.18), (7.19) are identical to those obtained by Sharma & Singh[79)] (for $S_1= 1$, $S_2=0$).

VI.3 SOLUTION OF THE PROBLEM

Substituting the expressions

$$G(\zeta,\tau) = \sum \in^N G_N(\zeta,\tau)$$

$$L(\zeta,\tau) = \sum \in^N L_N(\zeta,\tau)$$

$$H(\zeta,\tau) = \sum \in^N H_N(\zeta,\tau)$$

$$M(\zeta,\tau) = \sum \in^N M_N(\zeta,\tau)$$

$$\phi(\zeta,\tau) = \sum \in^N \phi_N(\zeta,\tau)$$

$$\Psi(\zeta,\tau) = \sum \in^N \Psi_N(\zeta,\tau)$$

(7.20)

into (7.14) to (7.19) neglecting the terms with coefficient of $\in^2$ (assumed negligible small) and equating the terms independent of $\in$ and coefficient of $\in$, we get the following equations:

$$G_0''= R\,\partial G_0/\partial\tau - \tau_1\,\partial G_0''/\partial\tau + m^2 G_0$$

(7.21)

$G_1'' = R\,\partial G_1/\partial\tau - 2R(H_0'G_0' - H_0G_0') - \tau_1\,\partial G_1''/\partial\tau - 2\tau_1(H_0'G_0''' - H_0''G_0') - 2\tau_2(H_0'G_0'' -$

$H_0''G_0') + m^2G_1$

(7.22)

$L_0'' = R\,\partial L_0/\partial\tau - \tau_1\,\partial L_0''/\partial\tau + m^2L_0$

(7.23)

$L_1'' = R\,\partial L_1/\partial\tau - 2R(M_0'G_0 - H_0L_0') - \tau_1\,\partial L_1''/\partial\tau - 2\tau_1(H_0'''L_0 + H_0''L_0' + H_0'L_0'' + H_0L_0'''$

$+2M_0''G_0' + 2M_0'G_0'') - 2\tau_2(H_0'''L_0 + H_0''L_0' + H_0'L_0'' + 2M_0''G_0' + M_0'G_0'') + m^2L_1$

(7.24)

$H_0^{iv} = R\partial H_0''/\partial\tau - \tau_1\partial H_0^{iv}/\partial\tau + m^2H_0''$

(7.25)

$H_1^{iv} = R\,\partial H_1''/\partial\tau + 2R(H_0H_0''' + G_0G_0') - \tau_1\,\partial H_1^{iv}/\partial\tau - 2\tau_1(H_0'H_0^{iv} + H_0H_0^{v} + 2H_0''H_0''' +$

$4G_0'G_0'') - 2\tau_2(3G_0'G_0'' + H_0'H_0^{iv} + 2H_0''H_0''') + m^2H_1''$

(7.26)

$M_0^{iv} = R\partial M_0''/\partial\tau - \tau_1\partial M_0^{iv}/\partial\tau + m^2M_0''$

(7.27)

$M_1^{iv} = R\partial M_1''/\partial\tau + 2R(H_0'M_0'' + H_0M_0''' - L_0'G_0 - L_0G_0') - \tau_1\partial M_1^{iv}/\partial\tau - 2\tau_1(2H_0'''M_0''$

$+H_0^{iv}M_0' + 2H_0''M_0''' - 4L_0'G_0'' - 2L_0''G_0' - 2L_0G_0''') + H_0M_0^{v} + 2H_0'M_0^{iv}) -$

$2\tau_2(2H_0'''M_0'' + H_0^{iv}M_0' + 2H_0''M_0''' - 3L_0'G_0'' - 2L_0''G_0' -$

$L_0G_0''' + H_0'M_0^{iv}) + m^2M_1$ (7.28)

$\Psi_0'' = RP_r\partial\Psi_0,$

(7.29)

$\Psi_1'' = RP_r[\partial\Psi_1 - 2H_0'\Psi_0 + 2H_0\Psi_0' - H_0''^2 - G_0'^2 - \tau_1(H_0''\partial H_0'' + G_0'\partial G_0'),$

(7.30)

$4\Psi_0 + \phi_0'' = RP_r\partial\Psi_0,$

(7.31)

$4\Psi_1 + \phi_0'' = RP_r[\partial\phi_1 + (R_m/R_z)2M_0'\Psi_0 + 2H_0\phi_0' - 12H_0'^2 + (R_m/R_z)2H_0''M_0'' - (R_L/R_z)2L_0'G_0' - \tau_1\{(12H_0'\partial H_0' - (R_m/R_z)(H_0''\partial M_0'' + M_0''\partial H_0'') + (R_L/R_z)(G_0'\partial L_0')\}].$ (7.32)

Taking $G_n(\zeta,\tau) = G_{ns}(\zeta) + e^{i\tau}G_{nt}(\zeta)$

$L_n(\zeta,\tau) = L_{ns}(\zeta) + e^{i\tau}L_{nt}(\zeta)$

$H_n(\zeta,\tau) = H_{ns}(\zeta) + e^{2i\tau}H_{nt}(\zeta)$

$M_n(\zeta,\tau) = M_{ns}(\zeta) + e^{2i\tau}M_{nt}(\zeta)$

$\phi_n(\zeta,\tau) = \phi_{ns}(\zeta) + e^{2i\tau}\phi_{nt}(\zeta)$

$\Psi_n(\zeta,\tau) = \Psi_{ns}(\zeta) + e^{2i\tau}\Psi_{nt}(\zeta)$

(7.33)

Complex notation has been adopted here with the convention that only real parts of the complex quantities have the physical meaning.

Using (7.24) and (7.33), the boundary conditions (7.5) & (7.7) for n = 0, 1 transform to

$G_{0s}(0) = 0, \quad G_{0t}(0) = 1, \qquad G_{1s}(0) = 0, \quad G_{1t}(0) = 0,$

$G_{0s}(1) = 0, \quad G_{0t}(0) = 0, \qquad G_{1s}(1) = 0, \quad G_{1t}(1) = 0,$

$H_{0s}(0) = k,\quad H_{0t}(0) = 0,\qquad H_{1s}(0) = 0,\quad H_{1t}(0) = 0,$

$H_{0s}(1) = k,\quad H_{0t}(1) = 0,\qquad H_{1s}(1) = 0,\quad H_{1t}(1) = 0,$

$H'_{0s}(0) = 0,\ H'_{0t}(0) = 0,\qquad H'_{1s}(0) = 0,\quad H'_{1t}(0) = 0,$

$H'_{0s}(1) = 0,\ H'_{0t}(1) = 0,\qquad H'_{1s}(1) = 0,\quad H'_{1t}(1) = 0,$

$L_{0s}(0) = 0,\quad L_{0t}(0) = 0,\qquad L_{1s}(0) = 0,\quad L_{1t}(0) = 0,$

$L_{0s}(1) = 0,\quad L_{0t}(1) = 0,\qquad L_{1s}(1) = 0,\quad L_{1t}(1) = 0,$

$M'_{0s}(0) = 0,\ M'_{0t}(0) = 0,\qquad M'_{1s}(0) = 0,\quad M'_{1t}(0) = 0,$

$M'_{0s}(1) = 0,\ M'_{0t}(1) = 0,\qquad M'_{1s}(1) = 0,\quad M'_{1t}(1) = 0,$

$M_{0s}(0) = 0,\quad M_{0t}(0) = 0,\qquad M_{1s}(0) = 0,\quad M_{1t}(0) = 0,$

$M_{0s}(0) = 0,\quad M_{0t}(0) = 0,\qquad M_{1s}(0) = 0,\quad M_{1t}(0) = 0,$

$M_{0s}(1) = 0,\quad M_{0t}(1) = 0,\qquad M_{1s}(1) = 0,\quad M_{1t}(1) = 0,$

$\Psi_{0s}(0) = 0,\quad \Psi_{0t}(0) = 0,\qquad \Psi_{1s}(0) = 0,\quad \Psi_{1t}(0) = 0,$

$\Psi_{0s}(1) = 0,\quad \Psi_{0t}(1) = 0,\qquad \Psi_{1s}(1) = 0,\quad \Psi_{1t}(1) = 0,$

$\phi_{0s}(0) = 0,\quad \phi_{0t}(0) = 0,\qquad \phi_{1s}(0) = 0,\quad \phi_{1t}(0) = 0,$

$\phi_{0s}(1) = 0,\quad \phi_{0t}(1) = 0,\qquad \phi_{1s}(1) = 0,\quad \phi_{1t}(1) = 0,$ (7.34)

Applying (7.33) & (7.34) in eqs. (7.21) to (.32), we get

$G_{0s}(\zeta) = G_{1s}(\zeta) = 0$,

$G_{0t}(\zeta) = C_1 e^{f\zeta} + C_1 e^{-f\zeta}$,

Where $C_1 = (1-e^{-f})/2\text{Sinh } f)$

$C_2 = 1-C_1$,

$f = \{(iR+m^2)/(1+i\tau)\}^{1/2} = A+iB,$

$A = [[(m^2+R\tau_1)+(m^2+R\tau_1)^2+(R-m^2\tau_1)^2\}]/\{2(1+\tau_1^2\}]^{1/2},$

$B = [[(m^2+R\tau_1)^2+(R-m^2\tau_1)^2\}-(m^2+R\tau_1)]/\{2(1+\tau_1^2\}]^{1/2},$

$G_0(\zeta,\tau) = \text{Real}\{e^{i\tau}G_{0t}(\zeta)\},$

$G_0(\zeta,\tau) = G_0(\zeta,\tau) + \in G_1(\zeta,\tau) = \text{Real}(e^{i\tau}\, G_{0t}(\zeta)\}$

$M_{0t}(\zeta) = M_{1t}(\zeta) = 0$

$M_{0s}(\zeta) = A_1 e^{m\zeta}/m^2 + A_2 e^{-m\zeta}/m^2 + A_3\zeta + A_4,$

$M'_{0s}(\zeta) = A_1 e^{m\zeta}/m - A_2 e^{-m\zeta}/m + A_3,$

Where $A_1 = A_2(e^{-m}-1)/(e^m-1),$

$A_2 = m^2(e^m-1)/\{4+e^m(m-2)-e^{-m}(m+2)\},$

$A_3 = -(A_1+A_2)/m,$

$A_4 = (A_1+A_2)/m^2.$

$Y_{36} = X_{51}+(X_{26}/2RP_r),$

$Y_{37} = X_{45}+(X_{22}/2RP_r),$

$Y_{38} = -X_{52}+(X_{25}/2RP_r),$

$Y_{39} = X_{46}-(X_{21}/2RP_r),$

$Y_{40} = X_{52}-(X_{25}/2RP_r),$

$Y_{41} = X_{51}+(X_{26}/2RP_r),$

$Y_{42} = X_{46}-(X_{21}/2RP_r),$

$Y_{43} = -X_{45}-(X_{22}/2RP_r),$

$Y_{44} = Y_{36} \in Cos2\tau - Y_{40} \in Sin2\tau,$

$Y_{45} = Y_{38} \in Cos2\tau - Y_{41} \in Sin2\tau,$

$Y_{46} = Y_{37} \in Cos2\tau - Y_{42} \in Sin2\tau,$

$Y_{47} = Y_{39} \in Cos2\tau - Y_{43} \in Sin2\tau,$

$Y_{48} = \{X_{25} + X_{26})/Q\} \in Cos2\tau - \{X_{26} - X_{25})/Q \in Sin2\tau,$

$Y_{49} = \{X_{25} - X_{26})/Q\} \in Cos2\tau - \{X_{26} + X_{25})/Q \in Sin2\tau,$

$Y_{50} = \{X_{21} + X_{22})/Q\} \in Cos2\tau - \{X_{22} - X_{21})/Q \in Sin2\tau,$

$Y_{51} = \{X_{22} - X_{21})/Q\} \in Cos2\tau - \{X_{22} + X_{21})/Q \in Sin2\tau,$

$Y_{52} = Y_{26} \in Cos2\tau - Y_{31} \in Sin2\tau,$

$Y_{53} = Y_{27} \in Cos2\tau - Y_{32} \in Sin2\tau,$

$Y_{54} = Y_{28} \in Cos2\tau - Y_{33} \in Sin2\tau,$

$Y_{55} = Y_{29} \in Cos2\tau - Y_{34} \in Sin2\tau,$

$Y_{56} = Y_{30} \in Cos2\tau - Y_{35} \in Sin2\tau,$

VII.4 RESULTS AND DISCUSSION

From fig(1) to fig(4), it is clear that the temperature decreases continuously with increase in ζ for all the values of elastic-viscous parameter τ_1, Reynolds number R at phase difference $\tau = \pi/3$ and 0 both.

The variation of the temperature distribution with ζ at $\xi = 5$, $\in = 0.02$, k = 1, R= 7, m = 10, P = 6, E = 5 for different values of $\tau_1 = 3, 4, 5$ when phase difference $\tau = \pi/3$ and 0 is shown in fig (1) and fig (2) respectively. It is clear from fig (1) that the temperature increases with an increase in τ_1 throughout the gap-length whenever this behaviour is reversed in fig (2).

from fig (1) that the temperature increases with an increase in τ_1 throughout the gap-length whenever this behaviour is reversed in fig (2).

The variation of the temperature distribution with ζ at $\xi = 5$, $\in = 0.02$, k = 1, $\tau_1 = 3$, m = 10, P = 6, E = 5 for different values of $R = 2, 3, 4$ when phase difference $\tau = \pi/3$ and 0 is shown in fig (3) and fig (4) respectively. It is evident from fig (3) that the temperature increases with an increase in Reynolds number R throughout the gap-length. It is seen from fig (4) that the temperature increases with an increase in Reynolds number R near the lower and upper disc whenever in the middle of the gap-length this trend of variation of temperature with R changes frequently.

The variation of the temperature distribution with ζ at $\xi = 5$, $\in = 0.02$, k = 1, $\tau_1 = 3$, P = 6, E = 5, R = 7 for different values of m = 1, 8, 15 when phase difference $\tau = \pi/3$ and 0 is shown in fig (5) and fig (6) respectively. It is seen from fig (5) that the temperature decreases continuously with an increase in ζ. It is also observed from fig (5) that the temperature decreases with an increase in magnetic field parameter m near the lower and upper disc but in the middle of the gap-length the branches of the temperature graph are very closed to each other whenever their trend of variation of temperature with magnetic field parameter m is approximately similar to that of near the lower and upper disc. It is evident from fig (6) that die m = 1, 8 temperature decreases continuously with an increase in ζ whenever for m = 15 temperature increases near the lower disc and decreases thereafter upto the surface of upper disc. It is also clear from this figure that the behaviour of temperature for different values of magnetic field parameter m is just reversed to that fig (5).

The variation of the temperature distribution with ζ at $\xi = 5$, $\in = 0.02$, m = 10, $\tau_1 = 3$, P = 6, E = 5, R = 7 for different values of $k = 1, 2, 3$ when phase difference $\tau = \pi/3$ and 0 is shown in fig (7) and fig (8) respectively. At k = 1, the temperature decreases throughout the gap-length whenever at k = 2 and k = 3, it increases near the lower disc and decreases rapidly thereafter upto the surface of the upper disc. It is also clear from fig (7) that the temperature increases with an increase in suction parameter k throughout the gap-length in fig (8), the behaviour of the temperature is similar to that of fig (7).

The variation of the Nusselt number Nu_a with ξ at E= 5, $\tau_1 = 5$, m = 10, R = 7, P = 6, $\in = 0.02$, $\xi_0 = 5$, $\tau = \pi/3$ for different values of suction parameter k = 1, 3, 5 is shown in fig (9). It is seen from this figure that the Nusselt number decreases with an increase in ξ for all the values of k. It is also evident from this figure that Nusselt number decreases with an increase in suction parameter k throughout the gap length.

The variation of the Nusselt number Nu_a with ξ at E= 5, $\tau_1 = 5$, m = 10, R = 10, P = 6, $\in = 0.02$, $\xi_0 = 5$, $\tau = \pi/3$ for different values of suction parameter k = 1, 3, 5 is shown in fig (10). It is clear from this figure that the Nusselt number increases with an increase in ξ for all the values of k. It is also observed from this figure that Nusselt number increases with an increase in suction parameter k throughout the gap length.

	$\tau = \pi/3$			$\tau = 0$		
ζ	$\tau_1 = 3$	$\tau_1 = 4$	$\tau_1 = 5$	$\tau_1 = 3$	$\tau_1 = 4$	$\tau_1 = 5$
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	0.968509	0.968509	0.980498	0.996076	0.964325	0.970621
0.2	0.935000	0.935000	0.946522	0.954568	0.902526	0.910906
0.3	0.883913	0.883913	0.892623	0.889206	0.820106	0.829575
0.4	0.812628	0.812628	0.819084	0.808085	0.725491	0.736101
0.5	0.722827	0.722827	0.727551	0.713782	0.625122	0.636649
0.6	0.615324	0.615324	0.618742	0.606725	0.521453	0.533340
0.7	0.488578	0.488578	0.491888	0.486714	0.412581	0.424382

0.8	0.340294	0.340294	0.345492	0.351409	0.293043	0.304244
0.9	0.172566	0.172566	0.179658	0.193232	0.156277	0.164810
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Table (1) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different elastic-viscous parameter τ			Table (2) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different elastic0-viscous parameter τ		

	$\tau = \pi/3$			$\tau = 0$		
ζ	$\tau_1 = 3$	$\tau_1 = 4$	$\tau_1 = 5$	$\tau_1 = 3$	$\tau_1 = 4$	$\tau_1 = 5$
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	0.834784	0.892147	0.918727	0.951933	0.964325	0.970621
0.2	0.721537	0.813781	0.855260	0.894243	0.902526	0.910906
0.3	0.635979	0.744163	0.791034	0.821796	0.820106	0.8299575
0.4	0.558557	0.669559	0.716339	0.735955	0.725491	0.736101
0.5	0.476417	0.582464	0.627234	0.639469	0.625122	0.636649
0.6	0.383708	0.479966	0.522402	0.533610	0.521453	0.533340
0.7	0.281206	0.362904	0.401886	0.417359	0.412581	0.424382
0.8	0.175553	0.236260	0.268192	0.288555	0.293043	0.304244
0.9	0.077759	0.110196	0.129091	0.147049	0.156277	0.164810
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Table (3) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different Reynolds number R			Table (4) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different Reynolds number R		

	$\tau = \pi/3$			$\tau = 0$		
ζ	m = 1	m = 8	m = 15	m = 1	m = 8	m = 15
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	0.982854	0.976766	0.911736	0.981948	0988950	1.033528
0.2	0.948199	0.942952	0.883227	0.939399	0.946571	0.995134
0.3	0.893605	0.889640	0.852853	0.878626	0.883420	0.915020
0.4	0.819975	0.816478	0.799494	0.801946	0.804705	0.818469
0.5	0.728594	0.725358	0.721344	0.709738	0.711693	0.716547
0.6	0.619974	0.617091	0.619036	0.601956	0.604447	0.611948

0.7	0.493599	0.490560	0.489310	0.478647	0.482927	0.502840
0.8	0.348183	0.343768	0.329261	0.339429	0.345832	0.379723
0.9	0.182832	0.177244	0.150939	0.181974	0.188063	0.220365
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Table (5) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different magnetic field parameter m			Table (6) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different magnetic field parameter m		

	$\tau = \pi/3$			$\tau = 0$		
ζ	k = 1	k = 2	k = 3	k = 1	k = 2	k = 3
0.0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.1	0.968509	1.051032	1.133554	0.996076	1.077954	1.159831
0.2	0.935000	1.082812	1.230625	0.954568	1.094126	1.233685
0.3	0.883913	1.077343	1.270773	0.889206	1.068056	1.246907
0.4	0.812628	1.032661	1.252693	0.808085	1.010188	1.212291
0.5	0.722827	0.951572	1.180317	0.713782	0.923628	1.133474
0.6	0.615324	0.835356	1.055389	0.606725	0.808828	1.010931
0.7	0.488578	0.6820085	0.875438	0.786714	0.665565	0.844415
0.8	0.340294	0.488107	0.635919	0.351409	0.490968	0.630527
0.9	0.172566	0.255088	0.337661	0.193232	0.275110	0.356987
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	Table (7) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different suction parameter m			Table (8) variation of temperature distribution $(T-T_b)/(T_a-T_b)$ at different suction parameter m		

	Nu_a			Nu_b		
ζ	k = 1	k = 3	k = 5	k = 1	k = 3	k = 5
0.0	-3.152272	-4.926532	-6.700793	25.522789	27.378542	29.234295
0.1	-3.282830	-5.057090	-6.831350	25.732852	27.588605	29.444358
0.2	-3.674503	-5.448763	-7.223023	26.363043	28.218796	30.074549
0.3	-4.327291	-6.101551	-7.875811	27.413360	29.269113	31.124866
0.4	-5.241195	-7.015455	-8.789715	28.883805	3.0739558	32.595311
0.5	-6.416213	-8.190474	-9.964734	30.774377	32.630129	34.485882

0.6	-7.852348	-9.626608	-11.40087	33.085075	34.940828	36.796581
0.7	-9.549597	-11.32386	-13.09812	35.815901	37.671654	39.527407
0.8	-11.50796	-13.28222	-15.05648	38.966854	40.822607	42.678360
0.9	-13.72744	-15.50170	-17.27596	42.537933	44.393686	46.249439
1.0	-16.20804	-17.98229	-19.75956	46.529140	48.384893	50.240646
	Table (9) variation of Nusselt number Nu_a at different suction parameter k			Table (10) variation of Nusselt number Nu_b at different suction parameter k		

Summary and Discussions

MHD is the study of the motion of the electrically conducting fluids in the presence of electric and magnetic fields. When a conducting fluid is under the influence of the electro-magnetic field, it behaves differently than without electromagnetic field. This is mainly because of Lorentz force, which is a cross product of electric field and magnetic field (Sir Flemming's right hand law). Even without the external electric field, flow pattern is altered due to the presence of strong magnetic field. Magnetic field and the motion of the conducting fluid particles generate electric current. This current and magnetic field interact with each other, and change the flow motion, with a chain reaction, all three fields (velocity, magnetic, electric) are interconnected and reveal very unique features.

Heat transfer is that science, which seeks to predict the energy transfer, which may take place between material bodies as a result of temperature difference. In the simplest of the terms, the discipline of heat transfer is concerned with only two things: temperature and flow of heat. Temperature represents the amount of thermal energy available, whereas heat flow represents the movement of thermal energy from one place to another place.

In our present problem, we here study the flow pattern of an incompressible second-order fluid between two parallel infinite discs in the presence of transverse magnetic field when one is rotating (called rotor) and other is at rest (called stator). A uniform injection is applied to the stator forming the subject matter of the paper. The Rotor coincides with the plane z = 0 and the stator coincides with the plane z = d. Here the dimensionless parameters $\tau_1(\mu_2/pd^2)$, $\tau_2(\mu_2/pd^2)$ govern the effects of elastic-viscosity and cross–viscosity, while the effect of the injection are governed by a non-dimensional parameter k $(=w_0/2d\Omega)$ where w_0 is the uniform suction velocity (negative for injection).

Chapter II <u>RESULTS AND CONCLUSION</u>

The variation of radial velocity for different elastic-viscous parameter τ_1 = -1.3, -2, -2.6; when cross-viscous parameter τ_1 = 10, injection parameter k = 5 Reynolds number R= 0.05, magnetic field m_1 = 5 is shown in fig (1). In this figure, the curve of radial velocity w.r.t ζ is bell shaped with maximum at $\zeta = 0.5$ approximately. It is also evident from this from this figure that the radial velocity decreases with increase in τ_1 from ζ = 0.0-0.28, then it begins increase with increases in τ_1 upto ζ = 0.72 and then decreases with increase in τ_1 from ζ = 0.8-0.95. The value of radial velocity is approximately equal at ζ =0.28 and ζ – 0.72 for all values of τ_1. The point of maxima is in the middle of the gap length for all values of elastic-viscous parameter τ_1.

Due to complexity of the differential equations and tedious calculations of the solutions of the solutions, no one has tried to solve the most practical problems of enclosed torsionally oscillating discs so far. The authors have considered the present problem of flow of a non-Newtonian second-order fluid over an enclosed torsionally oscillating disc in the presence of the magnetic field and calculated successfully the steady and unsteady part both of the flow functions. The flow functions are expanded in the powers of the amplitude $\in$ (assumed to be small) of the oscillations of the disc. The non-Newtonian effects are exhibited through two dimensionless parameters $\tau_1(=n\mu_{2/}\, n\mu_1)$ and $\tau_2(=n\mu_{3/}\, n\mu_1)$, where μ_1, μ_2, μ_3 are coefficient of Newtonian viscosity, elastic-viscosity and cross viscosity respectively, n being the uniform frequency of the oscillation.

Chapter III. <u>RESULTS AND DISCUSION</u>

The variation of the radial velocity with ζ at τ_2= 2, ξ = 5, R = 5, R_m= 0.05, R_L = 0.049, R_z = 2, m = 2 for different values of elastic-viscous parameter τ_1= 0, -0.3 and phase difference τ= π/3, 2π/3 is shown in fig (1). For τ= π/3, the radial velocity increases with an increase in ζnear the lower disc, attains its maximum value at ζ=0.2then start decreasing, attain its minimum value at ζ=0.8 and increases thereafter near the upper disc. It is clear that the radial velocity increases with an increase in τ_1 near the lower disc then start decreasing with an increase in τ_1 after the point of intersection near the upper disc. For τ =2π/3, the radial velocity increases with an increase in ζ and start decreasing thereafter at τ_1=0 whenever at τ_1 = -0.3 it decreases first, attains its minimum value at ζ = 0.1then start increasing, attains its maximum value at $\zeta =$ 0.7and decreases there after upto the surface of the upper disc. It is also seen from this figure that the radial velocity increases with an increase in τ_1 upto the middle of the gap-length and decreases thereafter with an increase in τ_1 upto the surface of the upper disc.

The authors have considered the present problem of heat transfer in the flow of a non-Newtonian second-order fluid over an enclosed torsionally oscillating discs with uniform suction and injection in the presence of the magnetic field and calculated successfully the steady and unsteady part both of the flow and energy functions. The flow and energy functions are expanded in the powers of the amplitude $\in$ (assumed to be small) of the oscillations of the disc. The non-Newtonian effects are exhibited through two dimensionless parameters τ_1 (=nμ_2/μ_1) and τ_2(=nμ_3/μ_1), where μ_1, μ_2, μ_3 are coefficient of Newtonian viscosity, elastic-viscosity and cross-viscosity respectively, n being the uniform frequency of the oscillation. The variation of temperature distribution with elastic-viscous parameter τ_1, cross –viscous parameter τ_2 (based on the relation τ_1 =a τ_2, where a = -0.2 as for 5.46% poly-iso- butylenes type solution in cetane at 30^{0}C (Markowiz[38]) Reynolds number R_1 magnetic field m, suction parameter k at different phase difference τ is shown graphically.

Chapter VI. RESULTS AND DISCUSSION

The variation of the temperature distribution with ζ at R = 7, P = 6, ζ = 5, and ϵ =0.02, k =15,m = 10, E = 5 for different values of τ_1= 1, 1.2, 3 when $\tau = \pi$ /3 and $2\pi/3$ is shown in fig (1) and fig (2) respectively. From fig (1) and fig (2), the graph of the temperature variation is parabolic with vertex downwards. It is also clear from these figures that the temperature is minimum at the middle of the gap-length and remains negative throughout the gap-length except near the surface of the lower disc. It is seen from fig (1) that temperature increases with an increase in elastic-viscous parameter τ in the first half and being overlapped in the second half of the gap-length. It is observed from fig(2) that the temperature decreases with an increase in τ_1 in the middle of the gap-length and is being overlapped thereafter.

The variation of the temperature distribution with ζ at τ_1= 5, P = 6, ζ = 5, ϵ =0.02, k =15,m = 10, E = 5 for different values of R = 1, 1.5, 2 when $\tau = \pi/3$ and $2\pi/3$ is shown in fig (3) and fig (4) respectively. From fig (3) and fig (4), the graph of the temperature variation is parabolic with vertex downwards. It is also evident from these figures that the temperature is minimum at the middle of the gap length and remains negative throughout the gap-length except near the surface of the lower disc. It is also. It is also clear from these that temperature decreases with an increase in Reynolds number R throughout the gap-length.

The purpose of the present paper is an attempt to study the heat transfer in the flow of a second-order fluid through a channel with porous walls under a transverse magnetic field by regular perturbation technique. The second-order effects on the temperature profile are illustrated graphically for different values of the Hartman and Reynolds number. The results are also obtained for the Newtonian fluid by taking the second-order parameter to be zero.

Chapter V. RESULTS AND DISCUSSIONS

(i) The values of the functions f_0, f_1 and f_2 are identical to those obtained by Sharma and Singh[76].

(ii) For $\tau_2 = 0$ the results are in good agreement with those obtained by Terril and Shrestha[73].

(iii) For S = 0 the results are matching with those obtained by Agarwal[75].

The variation of the temperature profile at P = 0.4, $\zeta = 0.4$, E = 1, $S_1 = 1$, $\tau_2 = -1$ for R = 0.01, 0.1, 1.0 is represented in fig (1). It is evident that for R = 0.1, temperature increases with ξ upto $\xi = 0.7$ approximately and thereafter decreases very slowly and attains its value 1 at the boundary wall $\xi = 1$. At the R = 1 the temperature graph is parabolic with vertex upward and attains its maximum value at the middle of the wall gap-length with minimum at the boundary wall $\xi = -1$. At R = 0.01, Temperature increases linearly throughout the wall gap-length with minimum at the boundary wall $\xi = -1$ and maximum at $\xi = 1$ It is also clear from this figure that the temperature increases with an increase in suction Reynolds number R.

The variation of the temperature profile at P = 0.4, $\zeta = 0.4$, E = 1, $S_1 = 1$, R = 1, for $\tau_2 = 0,\ 0.1,\ 1.0$ is represented in fig (2). It is evident from this figure that temperature graph is approximately parabolic with vertex upward and attains its maximum value at the middle of the wall gap-length with minimum at the boundary wall $\xi = -1$. It is also observed from this figure that the temperature decreases with an increase in cross-viscous second-order parameter τ_2.

The variation of the temperature profile at P = 0.4, $\zeta = 0.4$, E = 1, R = 1, $\tau_2 = -1$ for $S_1 = 0, 1, 2$ is represented in fig (3). It is seen from this figure that the temperature graph is approximately parabolic with vertex upward and attains its maximum value at the middle of the wall gap-length with minimum at the boundary wall $\xi = -1$. It is also observed from this figure that the temperature decreases with an increase Hartman number S_1.

The authors have considered the present problem of heat transfer in the flow of a non-Newtonian second-order fluid over an enclosed torsionally oscillating discs with uniform suction and injection in the presence of the magnetic field and calculated successfully the steady and unsteady part both of the flow and energy functions. The flow and energy functions are expanded in the powers of the amplitude $\in$ (assumed to be small) of the oscillations of the disc. The non-Newtonian effects are exhibited through two dimensionless parameters τ_1 ($= n\mu_2/\mu_1$) and τ_2($= n\mu_3/\mu_1$), where μ_1, μ_2, μ_3 are coefficient of Newtonian viscosity, elastic-viscosity and cross-viscosity respectively, n being the uniform frequency of the oscillation. The variation of temperature distribution with elastic-viscous parameter τ_1, cross –viscous parameter τ_2 (based on the relation $\tau_1 = a\,\tau_2$, where a = -0.2 as for 5.46% poly-iso- butylenes type solution in cetane at 30^0C (Markowiz[38]) Reynolds number R_1 magnetic field m, suction parameter k at different phase difference τ is shown graphically.

Chapter VI. RESULTS AND DISCUSSION

The variation of the temperature distribution with ζ at R = 7, P = 6, ζ = 5, and ϵ =0.02, k =15,m = 10, E = 5 for different values of τ_1= 1, 1.2, 3 when $\tau = \pi/3$ and $2\pi/3$ is shown in fig (1) and fig (2) respectively. From fig (1) and fig (2), the graph of the temperature variation is parabolic with vertex downwards. It is also clear from these figures that the temperature is minimum at the middle of the gap-length and remains negative throughout the gap-length except near the surface of the lower disc. It is seen from fig (1) that temperature increases with an increase in elastic-viscous parameter τ in the first half and being overlapped in the second half of the gap-length. It is observed from fig(2) that the temperature decreases with an increase in τ_1 in the middle of the gap-length and is being overlapped thereafter.

The non-Newtonian effects are exhibited through two dimensionless parameters $\tau_1(=n\mu_2/\mu_1)$ and $\tau_2(=n\mu_3/\mu_1)$, where μ_1, μ_2, μ_3 are coefficient of Newtonian viscosity, elastic-viscosity and cross viscosity. n being the uniform frequency of the oscillation. The variation of temperature distribution with elastic-viscous parameter τ_1, cross-viscous parameter τ_2 (based on the relation $\tau_1 = \alpha\tau_2$, where α = -0.2 as for 5.46% poly-iso- butylenes type solution in cetane at 30^0 C (Markowitz[38]) Reynolds number R, magnetic field m, suction parameter k at different phases difference τ is shown graphically.

Chapter VII. RESULTS AND DISCUSSION

From fig(1) to fig(4), it is clear that the temperature decreases continuously with increase in ζ for all the values of elastic-viscous parameter τ_1, Reynolds number R at phase difference $\tau = \pi/3$ and 0 both.

The variation of the Nusselt number Nu_a with ξ at E= 5, $\tau_1 = 5$, m = 10, R = 10, P = 6, $\in$ = 0.02, $\xi_0 = 5$, $\tau = \pi/3$ for different values of suction parameter k = 1, 3, 5 is shown in fig (10). It is clear from this figure that the Nusselt number increases with an increase in ξ for all the values of k. It is also observed from this figure that Nusselt number increases with an increase in suction parameter k throughout the gap length. ❋❋❋❋❋❋

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